

# Grade 11 Physics

## Study Guide

Formulas · worked examples · practice problems with the answer key

Built to Ontario's SPH3U. Also fits BC Physics 11  
and covers most of Alberta Physics 20.



6 modules · 43 formulas · 79 practice problems



The Exam Room · formula.expert · metric edition 1  
October 2026 · Mathieu Leclerc, with Claude

## About this study guide

**Who it's for.** Students working through Grade 11 Physics, and the teachers, tutors and parents printing handouts for them. Print it, copy it, staple it: it's free for classroom use, whole or by the page.

**How to use it.** Each module teaches its formulas in order (equation, picture, and what every symbol means), then closes with a practice set. Work the problems with a pencil; the answer key waits at the back so honest work gets an honest check. The two-page formula sheet up front is the exam-day companion, and every formula carries its number (Physics 11-1 onward) so your written work can cite exactly which one you used.

**Do it live.** Every problem in here has a living version in the Exam Room at [formula.expert/exams/grade-11-physics](https://formula.expert/exams/grade-11-physics): new numbers every attempt, hints that pop down when you want them, stars and streaks when you don't need them. When a printed problem stumps you, the matching lesson will coach you through it, step by step, free and with no sign-up. Other courses' study guides live at [formula.expert/study-guides](https://formula.expert/study-guides).

### Contents

|                                    |                           |       |
|------------------------------------|---------------------------|-------|
| The formula sheet                  | 43 formulas, with hints   | p. 4  |
| Module 1 · Kinematics              | 3 formulas · 12 problems  | p. 6  |
| Module 2 · Forces & Newton's laws  | 6 formulas · 12 problems  | p. 9  |
| Module 3 · Energy, work & power    | 7 formulas · 13 problems  | p. 14 |
| Module 4 · Waves & sound           | 13 formulas · 16 problems | p. 20 |
| Module 5 · Electricity & magnetism | 10 formulas · 14 problems | p. 29 |
| Module 6 · Heat                    | 4 formulas · 12 problems  | p. 38 |
| Answer key                         | problems 1–79             | p. 43 |

The practice problems are edition-drawn: a later edition deals fresh numbers, so keep the key with the printing it came from. The Exam Room deals new numbers every attempt.

## Figures

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**Fig. 1.1** Speed, Distance & Time Physics 11-1

**Fig. 1.2** Final Velocity (Uniform Acceleration) Physics 11-2

**Fig. 1.3** Displacement (Uniform Acceleration) Physics 11-3

**Fig. 2.1** Newton's Second Law Physics 11-4

**Fig. 2.2** Weight ( $W = mg$ ) Physics 11-5

**Fig. 2.3** Kinetic Friction Force ( $f = \mu_k N$ ) Physics 11-6

**Fig. 2.4** Maximum Static Friction ( $f = \mu_s N$ ) Physics 11-7

**Fig. 2.5** Hooke's Law Physics 11-8

**Fig. 2.6** Rope Tension When Lifting a Mass Physics 11-9

**Fig. 3.1** Work ( $W = Fd \cos \theta$ ) Physics 11-10

**Fig. 3.2** Kinetic Energy Physics 11-11

**Fig. 3.3** Gravitational Potential Energy ( $U = mgh$ ) Physics 11-12

**Fig. 3.4** Work–Energy Theorem Physics 11-13

**Fig. 3.6** Power from Force and Velocity ( $P = Fv$ ) Physics 11-15

**Fig. 3.7** Machine Efficiency Physics 11-16

**Fig. 4.1** Period-Frequency Relation Physics 11-17

**Fig. 4.2** Wave Speed ( $v = f\lambda$ ) Physics 11-18

**Fig. 4.3** Speed of Sound in Air Physics 11-19

**Fig. 4.4** Fundamental Frequency of a String Physics 11-20

**Fig. 4.5** Wave Speed on a String Physics 11-21

**Fig. 4.6** Harmonic Frequencies Physics 11-22

**Fig. 4.7** Fundamental of a Closed Pipe Physics 11-23

**Fig. 4.9** Sound Intensity ( $I = P/A$ ) Physics 11-25

**Fig. 4.10** Inverse-Square Law for Sound Physics 11-26

**Fig. 4.11** Doppler Effect (Approaching Source) Physics 11-27

**Fig. 4.12** Doppler Effect (Approaching Observer) Physics 11-28

**Fig. 4.13** Beat Frequency Physics 11-29

**Fig. 5.1** Electric Charge ( $Q = It$ ) Physics 11-30

**Fig. 5.2** Ohm's Law Physics 11-31

**Fig. 5.3** Two Resistors in Series Physics 11-32

**Fig. 5.4** Two Resistors in Parallel Physics 11-33

**Fig. 5.5** Electrical Power ( $P = VI$ ) Physics 11-34

**Fig. 5.6** Electrical Power ( $P = I^2R$ ) Physics 11-35

**Fig. 5.7** Electrical Power ( $P = V^2/R$ ) Physics 11-36

**Fig. 5.8** Electrical Energy ( $E = Pt$ ) Physics 11-37

**Fig. 5.9** Magnetic Force on a Current-Carrying Wire Physics 11-38

**Fig. 5.10** Transformer Voltage Ratio Physics 11-39

**Fig. 6.1** Sensible Heat ( $Q = mc\Delta T$ ) Physics 11-40

**Fig. 6.2** Latent Heat Physics 11-41

**Fig. 6.3** Heat Conduction Rate Physics 11-42

**Fig. 6.4** Thermal Linear Expansion Physics 11-43

## The formula sheet

### Module 1 • Kinematics

**Physics 11-1** Speed, Distance & Time

$$v = \frac{d}{t}$$

Average speed from distance travelled and elapsed time.

**Physics 11-2** Final Velocity (Uniform Acceleration)

$$v = v_0 + at$$

Final velocity after accelerating uniformly from an initial velocity for a given time.

**Physics 11-3** Displacement (Uniform Acceleration)

$$d = v_0 t + \frac{1}{2} at^2$$

Distance travelled under constant acceleration, starting from an initial velocity, over a time t.

### Module 2 • Forces & Newton's laws

**Physics 11-4** Newton's Second Law

$$F = ma$$

Force equals mass times acceleration.

**Physics 11-5** Weight ( $W = mg$ )

$$W = mg$$

Weight of a mass at Earth's surface, using standard gravity  $g = 9.80665 \text{ m/s}^2$  (exact by definition).

**Physics 11-6** Kinetic Friction Force ( $f = \mu_k N$ )

$$f_k = \mu_k N$$

Friction force resisting a sliding surface, equal to the coefficient of kinetic friction times the...

**Physics 11-7** Maximum Static Friction ( $f = \mu_s N$ )

$$f_{s,\max} = \mu_s N$$

Largest static friction force available before an object breaks loose and slides, from the static...

**Physics 11-8** Hooke's Law

$$F = kx$$

Restoring force of an ideal spring, proportional to its displacement from rest.

**Physics 11-9** Rope Tension When Lifting a Mass

$$T = m(g + a)$$

Tension in a rope lifting a mass with an upward acceleration, exceeding the static weight by the...

### Module 3 • Energy, work & power

**Physics 11-10** Work ( $W = Fd \cos \theta$ )

$$W = Fd \cos \theta$$

Work done by a constant force acting at an angle to the displacement.

**Physics 11-11** Kinetic Energy

$$E_k = \frac{1}{2} mv^2$$

Energy of a mass in motion.

**Physics 11-12** Gravitational Potential Energy ( $U = mgh$ )

$$U = mgh$$

Energy stored by raising a mass to height h near Earth's surface, with  $g = 9.80665 \text{ m/s}^2$ .

**Physics 11-13** Work–Energy Theorem

$$W = \frac{1}{2} m (v^2 - v_0^2)$$

Net work done on an object equals its change in kinetic energy, linking force and distance to a...

**Physics 11-14** Power ( $P = W/t$ )

$$P = \frac{W}{t}$$

Average power as work or energy delivered per unit time.

**Physics 11-15** Power from Force and Velocity ( $P = Fv$ )

$$P = Fv$$

Instantaneous power delivered by a force parallel to the velocity.

**Physics 11-16** Machine Efficiency

$$\eta = \frac{W_{out}}{W_{in}}$$

Efficiency of a machine as useful work out divided by work in, with the shortfall lost to friction,...

### Module 4 • Waves & sound

**Physics 11-17** Period-Frequency Relation

$$T = \frac{1}{f}$$

Period and frequency are reciprocals: seconds per cycle versus cycles per second.

**Physics 11-18** Wave Speed ( $v = f\lambda$ )

$$v = f\lambda$$

The universal wave equation: a wave's speed equals its frequency times its wavelength.

**Physics 11-19** Speed of Sound in Air

$$v = 331.3 + 0.606 T_C$$

The speed of sound in dry air grows about 0.6 m/s for every degree Celsius above freezing.

**Physics 11-20** Fundamental Frequency of a String

$$f = \frac{v}{2L}$$

Lowest standing-wave frequency of a string fixed at both ends.

**Physics 11-21** Wave Speed on a String

$$v = \sqrt{\frac{F}{\mu}}$$

Waves travel faster on a tighter, lighter string: speed is the square root of tension over linear...

**Physics 11-22** Harmonic Frequencies

$$f_n = n f_1$$

The nth harmonic of a vibrating system is n times its fundamental frequency.

**Physics 11-23** Fundamental of a Closed Pipe

$$f = \frac{v}{4L}$$

A pipe closed at one end resonates with a quarter wavelength inside, giving a fundamental of  $v/4L$ .

**Physics 11-24** Decibel Sound Level

$$\beta = 10 \log_{10} \left( \frac{I}{I_0} \right)$$

Sound level in decibels compares an intensity to the threshold of hearing,  $I_0 = 10^{-12} \text{ W/m}^2$ .

**Physics 11-25** Sound Intensity ( $I = P/A$ )

$$I = \frac{P}{A}$$

Sound intensity is the acoustic power passing through each square metre of surface.

**Physics 11-26** Inverse-Square Law for Sound

$$I = \frac{P}{4\pi r^2}$$

A point source's intensity falls with the square of distance as its power spreads over an expanding...

**Physics 11-27** Doppler Effect (Approaching Source)

$$f' = \frac{fv}{v - v_s}$$

Observed frequency rises when a sound source approaches, as each wavefront is emitted closer to the...

**Physics 11-28** Doppler Effect (Approaching Observer)

$$f' = \frac{f(v + v_o)}{v}$$

A listener moving toward a stationary source meets wavefronts more often and hears a higher...

**Physics 11-29** Beat Frequency

$$f_{\text{beat}} = f_1 - f_2$$

Two nearby tones interfere to produce a loudness pulse at their difference frequency, with  $f_1$  the...

### Module 5 • Electricity & magnetism

**Physics 11-30** Electric Charge ( $Q = It$ )

$$Q = It$$

Total charge transferred by a steady current flowing for a given time.

**Physics 11-31** Ohm's Law

$$V = IR$$

Relates the voltage across a conductor to the current through it and its resistance.

**Physics 11-32** Two Resistors in Series

$$R_t = R_1 + R_2$$

Total resistance of two resistors connected end to end is simply their sum.

**Physics 11-33** Two Resistors in Parallel

$$R_t = \frac{R_1 R_2}{R_1 + R_2}$$

Total resistance of two resistors connected side by side: product over sum, always less than either...

**Physics 11-34** Electrical Power ( $P = VI$ )

$$P = VI$$

Power delivered to a component as the product of the voltage across it and the current through it.

**Physics 11-35** Electrical Power ( $P = I^2 R$ )

$$P = I^2 R$$

Power dissipated as heat in a resistance carrying a current (Joule heating).

**Physics 11-36** Electrical Power ( $P = V^2/R$ )

$$P = \frac{V^2}{R}$$

Power dissipated in a resistance held at a fixed voltage.

**Physics 11-37** Electrical Energy ( $E = Pt$ )

$$E = Pt$$

Energy consumed by a device drawing constant power over a period of time.

**Physics 11-38** Magnetic Force on a Current-Carrying Wire

$$F = BIL \sin \theta$$

Magnitude of the force on a straight current-carrying wire in a uniform magnetic field.

**Physics 11-39** Transformer Voltage Ratio

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

Relates the primary and secondary voltages of an ideal transformer to its turns ratio.

**Module 6 · Heat****Physics 11-40** Sensible Heat ( $Q = mc\Delta T$ )

$$Q = mc\Delta T$$

Heat needed to change a mass's temperature: mass times specific heat times the temperature change.

**Physics 11-41** Latent Heat

$$Q = mL$$

Heat absorbed or released when a mass changes phase at constant temperature.

**Physics 11-42** Heat Conduction Rate

$$P = \frac{kA\Delta T}{d}$$

Steady-state heat flow through a slab by Fourier's law of conduction.

**Physics 11-43** Thermal Linear Expansion

$$\Delta L = \alpha L_0 \Delta T$$

Length change of a solid caused by a temperature change, via the linear expansion coefficient.

## MODULE 1

## Kinematics

Reading the problem · Choosing the tool · Speed, distance, time · Acceleration · Units in flight · Displacement · Two steps to the answer · The Final Bell

### 1.1 Speed, Distance & Time

(Physics 11-1)  $v = \frac{d}{t}$

Where,

$v$  = Speed (m/s)

$d$  = Distance (m)

$t$  = Time (s)

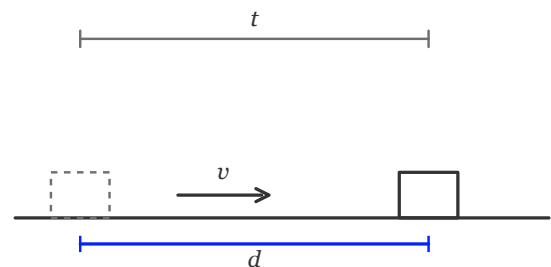


Figure 1.1: speed, distance & time.

Speed is ground covered divided by time taken,  $v = d/t$ , and it is worth being clear about what kind of statement that is. It is not a law of nature that could turn out to be false — it is the *definition* of average speed. Nothing in it can be wrong; it can only be misapplied. What it gives you is the single steady speed that would have covered the same distance in the same time, which is a genuinely useful summary of a trip and tells you almost nothing about any particular moment within it.

Take a drive of 148 km that takes 1 hour 45 minutes. Convert the time to a single unit first — 1.75 h — and  $v = 148/1.75 = 84.6$  km/h. In SI the same trip is 148 000 m over 6300 s, giving 23.5 m/s. Both answers describe the same drive, and the traffic light you sat at for ninety seconds is buried inside both of them.

This is the zeroth member of the kinematics family: set  $a = 0$  in  $d = v_0 t + \frac{1}{2} a t^2$  and you are left with  $d = vt$ . Run in the other direction, toward instantaneous speed, it becomes the derivative  $v = dd/dt$ , which is what your speedometer

reads and what calculus was partly invented to handle. The relation also underwrites the modern definition of length itself: since 1983 the metre has been defined by *fixing* the speed of light at 299 792 458 m/s exactly, so distance is now measured by timing light and rearranging this formula for  $d$ .

**The classic mistake is averaging the speeds instead of the trip.** Drive 60 km out at 60 km/h and return at 30 km/h, and the average for the round trip is not 45 km/h. The outbound leg takes 1 h, the return 2 h, so it is 120 km in 3 h — 40 km/h. Average speed is always total distance over total time, never the mean of the individual speeds, because you spend longer at the slow one. The second trap is units: this formula does not convert anything, so 100 km and 30 minutes gives 3.33 in units of km/min, not km/h. Fix the units before the arithmetic, or use the unit selectors on this page. And note that this is *distance*, the ground covered, not displacement — a runner who finishes a 400 m lap where she started has run at a respectable speed and has an average velocity of exactly zero.

**Worked example 1.1.** Given  $d = 100$  m,  $t = 8$  s; find  $v$  (speed). *100 m in 8 s → 12.5 m/s.*

## 1.2 Final Velocity (Uniform Acceleration)

(Physics 11-2)  $v = v_0 + at$

Where,

$v$  = Final velocity (m/s)

$v_0$  = Initial velocity (m/s)

$a$  = Acceleration (m/s<sup>2</sup>)

$t$  = Time (s)

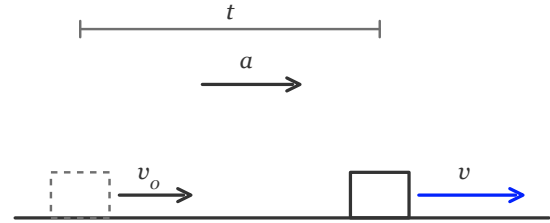


Figure 1.2: final velocity (uniform acceleration).

Under constant acceleration, velocity changes at a steady rate, so the final speed is simply the starting speed plus the acceleration multiplied by the elapsed time. Picture a car merging onto a highway: entering the ramp at 15 m/s and holding a steady 2 m/s<sup>2</sup> for 5 seconds, it reaches  $v = 15 + (2)(5) = 25$  m/s — right at highway pace. Deceleration works the same way with a negative  $a$ , which is how stopping times are estimated from braking data.

This is the first of the SUVAT equations, the toolkit of uniformly accelerated motion that traces back to Galileo's inclined-plane experiments in the early 1600s, where he showed that falling bodies gain equal speed in equal times. Because the relationship is linear in every variable, each of the four rearrangements has exactly one answer — no square roots, no ambiguity — making it the friendliest member of the kinematics family.

**Worked example 1.2.** Given  $v_0 = 15$  m/s,  $a = 2$  m/s<sup>2</sup>,  $t = 5$  s; find  $v$  (final velocity). *Car merging: 15 m/s + 2 m/s<sup>2</sup> for 5 s → 25 m/s.*

## 1.3 Displacement (Uniform Acceleration)

(Physics 11-3)  $d = v_0t + \frac{1}{2}at^2$

Where,

$d$  = Displacement (m)

$v_0$  = Initial velocity (m/s)

$a$  = Acceleration (m/s<sup>2</sup>)

$t$  = Time (s)

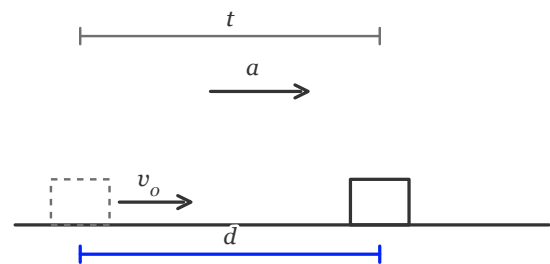


Figure 1.3: displacement (uniform acceleration).

When acceleration is constant, displacement has two parts: the distance you would cover at your initial velocity alone, plus the extra distance contributed by speeding up — and that extra grows with the *square* of time. A jet starting its takeoff roll from rest and holding 2 m/s<sup>2</sup> covers  $d = 0 + \frac{1}{2}(2)(30^2) = 900$  m in 30 seconds, which is why runways are measured in kilometres.

The  $\frac{1}{2}$  appears because the acceleration term is built from the average of a speed that grows linearly from zero. Galileo uncovered the underlying pattern — distances in successive equal time intervals follow the odd numbers 1, 3, 5, 7 — by rolling bronze balls down inclined planes. Note that solving for  $t$  would mean solving a quadratic with potentially two positive roots, so this calculator rearranges only for  $d$ ,  $v_0$ , and  $a$ , where the answer is always single-valued.

**Worked example 1.3.** Given  $v_0 = 0 \text{ m/s}$ ,  $a = 2.5 \text{ m/s}^2$ ,  $t = 30 \text{ s}$ ; find  $d$  (displacement). *Plane from rest,  $2.5 \text{ m/s}^2$  for  $30 \text{ s} \rightarrow 1125 \text{ m}$ .*

## Module 1 practice

1. Speed, distance, time: A delivery drone flies in a straight line at a constant  $11 \text{ m/s}$  for  $5.0 \text{ s}$ . *Determine the distance the drone travels.*
2. Speed, distance, time: A delivery drone flies in a straight line at a constant  $9 \text{ m/s}$  for  $5.0 \text{ s}$ . *Determine the distance the drone travels.*
3. Acceleration: A car accelerates uniformly at  $2.5 \text{ m/s}^2$  for  $8.0 \text{ s}$ . *Determine the increase in the car's speed.*
4. Acceleration: A car accelerates uniformly at  $2 \text{ m/s}^2$  for  $9.0 \text{ s}$ . *Determine the increase in the car's speed.*
5. Units in flight: A regional train is timed over a  $40 \text{ m}$  measured stretch, crossing it in  $4.0 \text{ s}$  at a constant speed. *Calculate the train's speed in  $\text{km/h}$ .*
6. Units in flight: A scooter holds a steady  $54 \text{ km/h}$  along a bike lane for  $10.0 \text{ s}$ . *Determine the distance the scooter covers.*
7. Displacement: A skateboarder rolls past a line at  $3 \text{ m/s}$  and accelerates uniformly down the slope at  $1 \text{ m/s}^2$ . *Determine how far she travels in the next  $4.0 \text{ s}$ .*
8. Displacement: A rower covers a  $60 \text{ m}$  course in  $6.0 \text{ s}$ , accelerating uniformly at  $1 \text{ m/s}^2$  the whole way. *Calculate the rower's speed at the START of the run.*
9. Two steps to the answer: A delivery scooter pulls away from rest, gaining speed steadily until it reaches  $12 \text{ m/s}$  after  $6.0 \text{ s}$ . It then holds that speed for a further  $8.0 \text{ s}$ . *Determine the total distance covered, first push to the end.*
10. Two steps to the answer: A cyclist rolling at  $8 \text{ m/s}$  spots the crossing guard and brakes smoothly to a stop over  $2.0 \text{ s}$ . *Determine the cyclist's acceleration, and how far the bike travels while stopping.*
11. The Final Bell: Last question of the paper. A courier bike starts from rest and accelerates uniformly at  $3 \text{ m/s}^2$  for  $4.0 \text{ s}$ , holds its speed for  $5.0 \text{ s}$ , then brakes smoothly to a stop in  $2.0 \text{ s}$ . Work each line — every answer feeds the next. *Determine the speed reached, then each distance, then the whole run.*
12. The Final Bell: Bonus mark, worked backwards: skid marks show a streetcar rolled to rest over  $30 \text{ m}$ , and the event recorder shows the braking took  $6.0 \text{ s}$ . *Determine how fast the streetcar was moving when the brakes went on.*

## MODULE 2

## Forces & Newton's laws

Reading the forces · Choosing the force tool ·  $F = ma$ , asked every way · Weight is not mass · Friction holds on · The spring pushes back · Unbalanced · The Loading Dock

### 2.1 Newton's Second Law

(Physics 11-4)  $F = ma$

Where,

$F$  = Force (N)

$m$  = Mass (kg)

$a$  = Acceleration ( $\text{m/s}^2$ )

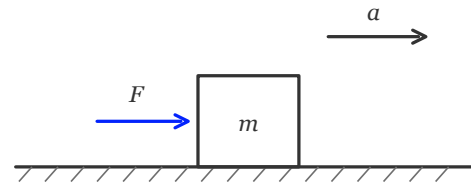


Figure 2.1: newton's second law.

Newton's second law says that the acceleration of an object is proportional to the net force on it and inversely proportional to its mass. Push twice as hard and it speeds up twice as fast; make it twice as heavy and it speeds up half as fast. The equation also defines the unit: one newton is exactly the force that accelerates one kilogram at one metre per second squared, so  $1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$ . Newton himself did not write  $F = ma$ ; he wrote that force is the rate of change of momentum,  $F = dp/dt$ , which is the more general statement and reduces to  $ma$  whenever the mass is not changing.

A 1400 kg car reaching 100 km/h — that is 27.8 m/s — in 8.5 s has an average acceleration of  $a = 27.8/8.5 = 3.27 \text{ m/s}^2$ , so the net force driving it forward is  $F = 1400 \times 3.27 \approx 4580 \text{ N}$ . The engine has to supply more than that, because drag and rolling resistance are pulling the other way and the 4580 N is what is left over after they have taken their share.

Almost every other force relation on this site is this one wearing a hat. Weight is  $F = ma$  with  $a = g$ . Centripetal force is  $F = ma$  with  $a = v^2/r$ . Impulse is this law

integrated over time, and the work–energy theorem is it integrated over distance. Its rotational twin,  $\tau = I\alpha$ , swaps torque for force and moment of inertia for mass and behaves identically.

**The word doing the most work in the law is "net", and it is the word most often dropped.** A crate being pushed with 200 N across a floor that resists with 150 N of friction accelerates as though 50 N were acting, not 200. If an object moves at constant speed the net force on it is zero, however many forces are actually pushing on it. The second mistake is arithmetically worse: **kilograms are not a force.** A "70 kg load" is a mass, and the force it exerts on its hanger is  $70 \times 9.81 \approx 687 \text{ N}$ . Feeding 70 into this calculator as a force when you meant a mass is wrong by a factor of 9.81. The imperial world hides the same trap behind identical words — the pound-mass and the pound-force are different quantities related by  $32.174 \text{ ft/s}^2$ , which is where the notorious  $g_c$  conversion factor comes from. Use the unit selectors and let them handle it.

**Worked example 2.1.** Given  $m = 70 \text{ kg}$ ,  $a = 1 \text{ g}$ ; find  $F$  (force). *70 kg under standard gravity  $\rightarrow 686.4655 \text{ N}$ .*

## 2.2 Weight ( $W = mg$ )

(Physics 11-5)  $W = mg$

Where,

$W$  = Weight (N)

$m$  = Mass (kg)

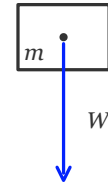


Figure 2.2: weight ( $w = mg$ ).

**Mass and weight are different quantities, and this formula is the exchange rate between them.** Mass is how much matter an object contains, measured in kilograms, and it is the same everywhere in the universe. Weight is the gravitational force acting on that mass, measured in newtons, and it changes with where you are standing. The relation  $W = mg$  is just Newton's second law with the acceleration set to whatever gravity supplies locally.

A 70 kg person weighs  $70 \times 9.80665 \approx 687$  N on Earth. Take the same person to the Moon, where  $g = 1.62$  m/s<sup>2</sup>, and they weigh about 113 N — roughly a sixth — while still being made of exactly 70 kg of person. On Mars,  $g = 3.72$ , the answer is 260 N. Nothing about the body changed; only the field it sits in did.

The  $g$  used here, 9.80665 m/s<sup>2</sup>, is *standard gravity*, a defined constant fixed by the third General Conference on Weights and Measures in 1901 so that engineering calculations would have one agreed number. Real gravity varies: about 9.780 m/s<sup>2</sup> at the equator and 9.832 at the poles, a 0.5% spread caused by the Earth's rotation and its

slightly flattened shape, with smaller local variations from altitude and the density of the rock underfoot. This whole formula is a near-surface shortcut for universal gravitation — put Earth's mass and radius into  $g = GM/R^2$  and 9.8 is what comes out.

**The everyday confusion is baked into the instruments.** A bathroom scale measures force, using a spring or a load cell, then divides by a value of  $g$  programmed at the factory and displays the result as kilograms. Take that scale to the Moon and it will confidently report that you have lost five-sixths of your mass. A balance, which compares your weight against known masses, would read correctly anywhere — which is the older and more honest instrument. The same confusion lives in the kilogram-force, a legacy unit equal to 9.80665 N, and in the imperial pound, which does double duty as a mass and a force. One more, because it comes up constantly: astronauts on the space station are not weightless because gravity has run out. At 400 km altitude  $g$  is still about 8.7 m/s<sup>2</sup>, nearly 89% of its surface value. They float because they and the station are both in free fall around the Earth together — falling, and missing.

**Worked example 2.2.** Given  $m = 70$  kg; find  $W$  (weight). *70 kg person  $\rightarrow$  686.4655 N.*

## 2.3 Kinetic Friction Force ( $f = \mu_k N$ )

(Physics 11-6)  $f_k = \mu_k N$

Where,

$f_k$  = Kinetic friction force (N)

$\mu_k$  = Coefficient of kinetic friction

$N$  = Normal force (N)

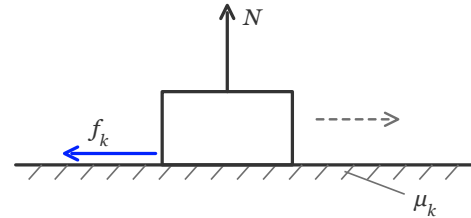


Figure 2.3: kinetic friction force ( $f = \mu_k n$ ).

Once a surface is already sliding, friction settles to a nearly constant value proportional to how hard the surfaces are pressed together:  $f = \mu_k N$ . Push a 200 N-loaded crate across a floor with  $\mu_k = 0.3$  and it resists with 60 N no matter how fast you shove it. The startling part — that friction depends on load but essentially *not* on contact area or speed — was established by Guillaume Amontons in 1699 and confirmed in Charles-Augustin de Coulomb's prize-winning 1785 study of rope, axles and rigging for the French navy, work so thorough that dry friction is still called Coulomb friction.

Typical coefficients: rubber on dry asphalt around 0.7, steel on steel about 0.6 lubricated down to 0.05, waxed ski on snow near 0.05, and PTFE on steel about 0.04. The trap is assuming  $N$  equals the weight — true only on level ground with no extra push or pull. On a slope  $N$  shrinks to  $mg \cos \theta$ , and a downward-angled push or a car's aerodynamic downforce raises it. Note also that kinetic friction always runs slightly below the static maximum, which is why a heavy box lurches forward the instant it breaks free.

**Worked example 2.3.** Given  $\mu_k = 0.3$ ,  $N = 200$  N; find  $f_k$  (kinetic friction force).  $\mu_k 0.3$  on  $200$  N normal  $\rightarrow 60$  N.

## 2.4 Maximum Static Friction ( $f = \mu_s N$ )

(Physics 11-7)  $f_{s,\max} = \mu_s N$

Where,

$f_s$  = Maximum static friction (N)

$\mu_s$  = Coefficient of static friction

$N$  = Normal force (N)

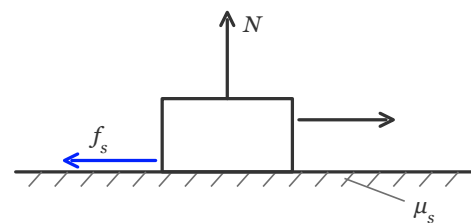


Figure 2.4: maximum static friction ( $f = \mu_s n$ ).

Static friction is the only force in introductory mechanics that is written with an inequality:  $f \leq \mu_s N$ . It supplies exactly whatever is needed to prevent sliding, up to a ceiling of  $\mu_s N$ , and this formula computes that ceiling. Press a 500 N normal load onto a surface with  $\mu_s = 0.6$  and you can push with anything up to 300 N and nothing moves; at 301 N the object breaks free and the weaker kinetic friction takes over.

That inequality is the single biggest trap: plugging  $\mu_s N$  in as "the friction force" on a stationary object is wrong unless the object is on the verge of slipping. The distinction pays real dividends — a car's tyres grip through static friction as long as they roll, which is why ABS pumps the brakes to keep them from locking into a lower- $\mu_k$  skid, and why a driven wheel spinning on ice suddenly has far less traction than one that is merely rolling.

**Worked example 2.4.** Given  $\mu_s = 0.6$ ,  $N = 500 \text{ N}$ ; find  $f_s$  (maximum static friction).  $\mu_s$  0.6 on 500 N normal  $\rightarrow$  300 N.

## 2.5 Hooke's Law

(Physics 11-8)  $F = kx$

Where,

$F$  = Spring force (N)

$k$  = Spring constant (N/m)

$x$  = Displacement from rest (m)

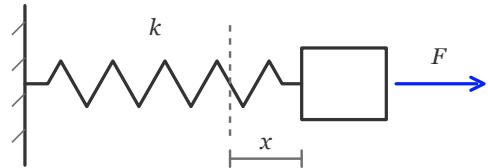


Figure 2.5: hooke's law.

Robert Hooke published this law in 1676 as a Latin anagram — *ceiiinosssttuv* — unscrambled two years later to "ut tensio, sic vis": as the stretch, so the force. An ideal spring pushes or pulls back in proportion to how far you displace it from rest. The spring constant  $k$  is the stiffness: a 200 N/m spring stretched 0.1 m pulls back with 20 N, while a car's suspension spring might run tens of thousands of N/m.

This calculator uses the magnitude form; strictly the restoring force points *opposite* the displacement, which is written  $F = -kx$  and is what makes released springs oscillate. The law holds only up to the elastic limit — stretch a spring too far and it deforms permanently. Within that limit it underpins spring scales, force gauges, vehicle suspensions, and even the atomic bonds that make solids springy.

**Worked example 2.5.** Given  $k = 200 \text{ N/m}$ ,  $x = 0.1 \text{ m}$ ; find  $F$  (spring force).  $200 \text{ N/m}$  stretched  $0.1 \text{ m} \rightarrow 20 \text{ N}$ .

## 2.6 Rope Tension When Lifting a Mass

(Physics 11-9)  $T = m(g + a)$

Where,

$T$  = Rope tension (N)

$m$  = Mass (kg)

$a$  = Upward acceleration ( $\text{m/s}^2$ )

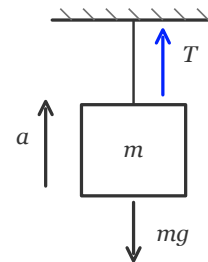


Figure 2.6: rope tension when lifting a mass.

Newton's second law on a hoisted load reads  $T - mg = ma$ , so the rope carries  $T = m(g + a)$ . Lift a 50 kg crate while accelerating upward at  $2 \text{ m/s}^2$  and the rope feels  $50 \times (9.80665 + 2) \approx 590 \text{ N}$ , about 20% more than the 490 N it holds at rest or at constant speed. Decelerate on the way up — or accelerate downward — and  $a$  goes negative, easing the tension; at  $a = -g$  the rope goes completely slack and the load is in free fall.

This is exactly why you feel heavy as an elevator starts up and light as it starts down: the floor is the "rope", and the scale under your feet reads  $m(g + a)$ . Rigging engineers turn the same relation into a dynamic load factor, sizing slings and hooks for the accelerating case rather than the static weight — snatching a load, or an emergency stop, can spike the tension well past the crane's nameplate figure.

**Worked example 2.6.** Given  $m = 50 \text{ kg}$ ,  $a = 2 \text{ m/s}^2$ ; find  $T$  (rope tension). *50 kg hoisted at  $2 \text{ m/s}^2 \rightarrow 590.33 \text{ N}$ .*

## Module 2 practice

**13.**  $F = ma$ , asked every way: A net force of 10 N acts on a 5 kg curling stone as it is driven out of the hack. *Calculate the acceleration the force produces.*

**14.**  $F = ma$ , asked every way: During a floor test, a net force of 40 N accelerates a loaded pallet uniformly at  $4 \text{ m/s}^2$ . *Determine the pallet's mass.*

**15.** Weight is not mass: A 10 kg bag of cement hangs from a crane hook, perfectly still. *Calculate the bag's weight.*

**16.** Weight is not mass: A 15 kg bag of cement hangs from a crane hook, perfectly still. *Calculate the bag's weight.*

**17.** Friction holds on: A mover slides a 30 kg crate across a level floor at a steady pace. The coefficient of kinetic friction between crate and floor is 0.4. *Determine the friction force on the crate.*

**18.** Friction holds on: Keeping a filing cabinet sliding at a constant speed takes a 60 N push. The floor presses up on the cabinet with a normal force of 200 N. *Determine the coefficient of kinetic friction.*

**19.** The spring pushes back: Stretching a trampoline spring 10 cm takes a steady 45 N pull. *Determine the spring constant.*

**20.** The spring pushes back: Stretching a trampoline spring 10 cm takes a steady 35 N pull. *Determine the spring constant.*

**21.** Unbalanced: A warehouse cart of mass 40 kg is pushed along the floor with a 100 N force while friction drags backward at 60 N. *Determine the cart's acceleration, net force first.*

**22.** Unbalanced: A crane cable lifts a 10 kg crate with a tension of 118 N, and the crate gains speed on the way up. *Determine the crate's upward acceleration, net force first.*

**23.** The Loading Dock: Last crate of the shift. A 20 kg crate sits on the loading dock,  $\mu_k = 0.3$  between crate and dock, and a worker pushes it with a steady 100 N. ( $g = 10 \text{ m/s}^2$  today.) Work each line — every answer feeds the next. *Determine the crate's speed after 4.0 s of pushing, one law at a time.*

**24.** The Loading Dock: Bonus mark, worked backwards: a parcel leaves a worker's hands sliding at 6 m/s along the dock, and friction alone brings it to rest in 2.0 s. ( $g = 10 \text{ m/s}^2$ .) *Determine the coefficient of kinetic friction between parcel and dock.*

## MODULE 3

## Energy, work & power

Paying in joules · Work is a push through a distance · The v-squared surprise · Height is stored energy · The energy swap · How fast is the work done · Nothing is perfect · The Drop Tower

### 3.1 Work ( $W = Fd \cos \theta$ )

(Physics 11-10)  $W = Fd \cos \theta$

Where,

$W$  = Work (J)

$F$  = Force (N)

$d$  = Displacement (m)

$\theta$  = Angle between force and motion ( $^\circ$ )

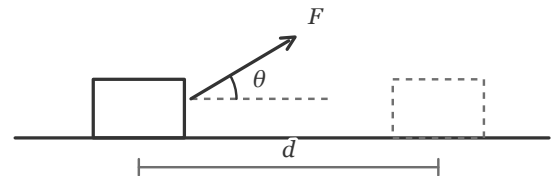


Figure 3.1: work ( $w = fd \cos \theta$ ).

In physics, work is force applied through a distance — and only the component of force *along* the motion counts, which is where the  $\cos \theta$  comes from. Pull a sled with 100 N on a rope angled  $30^\circ$  above the snow for 20 m, and you do  $W = 100 \times 20 \times \cos 30^\circ \approx 1732$  J, not the full 2000 J. The term itself was coined by the French engineer Gaspard-Gustave de Coriolis in 1826, precisely to compare what steam engines and horses could deliver.

Two useful edge cases: a force perpendicular to the motion ( $\theta = 90^\circ$ ) does no work at all — the Moon's orbit costs gravity nothing — and a force opposing the motion, like friction, does **negative** work, showing up as  $\cos \theta$  below zero. Solving for  $\theta$  uses the arccos principal branch,  $0^\circ$  to  $180^\circ$ , which conveniently covers the entire physical range of angles between two directions.

**Worked example 3.1.** Given  $F = 50$  N,  $d = 10$  m,  $\theta = 60^\circ$ ; find  $W$  (work). *50 N over 10 m at 60 deg  $\rightarrow$  250 J.*

### 3.2 Kinetic Energy

(Physics 11-11)  $E_k = \frac{1}{2}mv^2$

Where,

$E_k$  = Kinetic energy (J)

$m$  = Mass (kg)

$v$  = Speed (m/s)

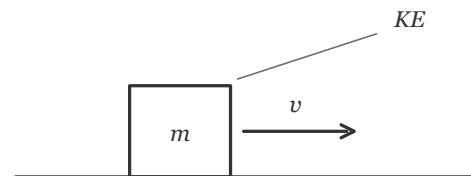


Figure 3.2: kinetic energy.

Kinetic energy is the energy an object carries because it is moving,  $E_k = \frac{1}{2}mv^2$ , and equivalently it is the work you would have to do to bring it from rest up to that speed — or

the work it can do on something else in coming back to rest. The two odd-looking features, the half and the square, both fall out of that second statement. Push with force  $F = ma$

through a distance and integrate:  $\int ma \, dx = \int mv \, dv = \frac{1}{2}mv^2$ . The square is not a modelling choice, it is what the integral hands back.

A 1500 kg car at 50 km/h — 13.9 m/s — carries  $\frac{1}{2} \times 1500 \times 13.9^2 \approx 145$  kJ. The same car at 100 km/h, 27.8 m/s, carries about 580 kJ. Twice the speed, **four** times the energy, and since the brakes can only dissipate energy at roughly a fixed rate per metre of road, roughly four times the distance to stop.

Which form of energy is the "real" one was a genuine dispute. Descartes and his followers backed  $mv$ ; Leibniz argued for what he called *vis viva*,  $mv^2$ . Willem 's Gravesande settled the experimental half of it in the 1720s by dropping brass balls into soft clay and finding that a ball arriving twice as fast sank about four times as deep, and Émilie du Châtelet made the theoretical case in the 1740s alongside her translation and commentary on the *Principia*.

Both quantities turned out to matter — momentum  $mv$  is conserved in every collision, kinetic energy only in elastic ones — which is why this site has pages for each.

**Everything that goes wrong here goes wrong at the square.** Doubling the speed does not double the energy, and the intuition that it does is what makes highway speeds feel deceptively similar to city ones. The unit trap follows directly: enter a speed in km/h where the formula wants m/s and you are wrong by  $3.6^2 = 12.96$ , not by 3.6 — the error is an order of magnitude and it looks plausible. Two smaller ones. Kinetic energy is a scalar with no direction, so two cars closing head-on do not have "negative" energy relative to each other, and you cannot cancel them the way you cancel momenta. And it is frame-dependent: a coffee cup on a train table has zero kinetic energy in your frame and a great deal in the frame of the platform. That is not a flaw; it is why the work–energy theorem only ever deals in *changes*.

**Worked example 3.2.** Given  $m = 2$  kg,  $v = 3$  m/s; find  $E_k$  (kinetic energy). *2 kg at 3 m/s  $\rightarrow$  9 J.*

### 3.3 Gravitational Potential Energy ( $U = mgh$ )

(Physics 11-12)  $U = mgh$

Where,

$U$  = Potential energy (J)

$m$  = Mass (kg)

$h$  = Height (m)

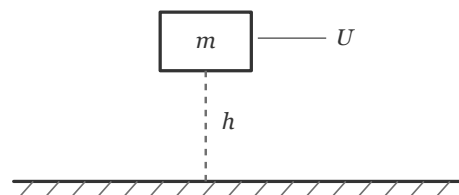


Figure 3.3: gravitational potential energy ( $U = mgh$ ).

Lifting a mass banks energy in the gravitational field, and near Earth's surface the deposit is simply  $mgh$ , with  $g = 9.80665$  m/s<sup>2</sup>. A roller coaster earns its entire ride on the first climb: a 500 kg car hauled 30 m up stores  $500 \times 9.80665 \times 30 \approx 147$  kJ, which the drops and loops then spend as speed. Only *differences* in height matter — you are free to call the ground floor, the table top, or sea level "zero", as long as you stay consistent.

The same idea runs entire power grids: pumped-storage hydro plants push water uphill when electricity is cheap and let it fall through turbines at peak demand, storing gigawatt-hours as nothing more than elevated water. The formula is a near-surface approximation — it treats  $g$  as constant, excellent for heights small compared with Earth's radius.

**Worked example 3.3.** Given  $m = 2$  kg,  $h = 10$  m; find  $U$  (potential energy). *2 kg lifted 10 m  $\rightarrow$  196.133 J.*

### 3.4 Work–Energy Theorem

(Physics 11-13)  $W = \frac{1}{2}m(v^2 - v_0^2)$

Where,

$W$  = Net work (J)

$m$  = Mass (kg)

$v$  = Final speed (m/s)

$v_0$  = Initial speed (m/s)

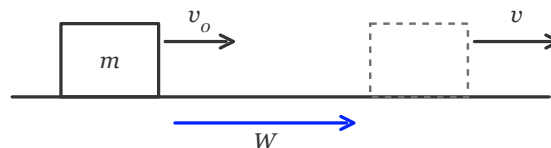


Figure 3.4: work–energy theorem.

Whatever the forces, the *net* work done on an object shows up entirely as a change in its kinetic energy:  $W = \frac{1}{2}m(v^2 - v_0^2)$ . Accelerate a 2 kg mass from rest to 10 m/s and exactly 100 J went in, no matter whether it took 1 m of huge force or 100 m of gentle push. Gaspard-Gustave de Coriolis formalised both "work" and the  $\frac{1}{2}mv^2$  form of kinetic energy in 1829, precisely so factory owners could compare what different machines actually delivered.

The theorem's power is that it skips time entirely, making it the fastest route to braking distances: a 1360 kg car slowing from 26.8 m/s to 13.4 m/s sheds about 367 kJ, and dividing that by the braking force gives the stopping distance directly. Sign discipline is the trap — friction and braking do *negative* work, so  $W$  comes out negative whenever the object slows, and this calculator will happily return a negative number to tell you so.

**Worked example 3.4.** Given  $m = 2$  kg,  $v = 10$  m/s,  $v_0 = 0$  m/s; find  $W$  (net work). *2 kg from rest to 10 m/s → 100 J.*

### 3.5 Power ( $P = W/t$ )

(Physics 11-14)  $P = \frac{W}{t}$

Where,

$P$  = Power (W)

$W$  = Work or energy (J)

$t$  = Time (s)

Power is the rate at which work is done,  $P = W/t$ . It answers a different question from work: not "how much energy did this take?" but "how fast was it delivered?" Two apprentices carrying identical toolboxes up the same stairs do exactly the same work against gravity, and the one who takes the stairs two at a time develops more power. One watt is one joule per second, which makes the watt a small unit — a person working steadily manages perhaps 75 W, and a fit cyclist holds around 250 W for an hour.

Take 20 kg of tools hauled 12 m up a ladder. The work is  $mgh = 20 \times 9.80665 \times 12 \approx 2354$  J regardless of how it is done. Take 40 s over it and  $P = 2354/40 \approx 59$  W.

Rush it in 15 s and the same job demands 157 W. The energy bill is identical; only the rate has changed, and it is the rate that decides whether a motor is big enough.

James Watt coined horsepower in the 1780s as a sales tool. He needed to tell mill owners how many horses one of his engines would replace, measured a horse turning a mill wheel, and settled on 33 000 foot-pounds per minute — about 745.7 W. It was a marketing unit before it was an engineering one, and it has outlived the argument it was built to win. The same rate appears in two other forms on

this site:  $P = Fv$  when the work is a force moving something along, and  $P = \tau\omega$  when it is a torque turning a shaft.

**The commonest error is treating power as though it were energy.** A kilowatt-hour is not a unit of power — it is a power multiplied by a time, so it is an *energy*, equal to 3.6 MJ. A 100 W bulb does not consume "100 watts per hour"; it consumes 100 watts, which over an hour amounts to 0.1 kWh. The phrase "watts per hour" is almost always a symptom that the two ideas have been mixed. Second, watch which horsepower a figure is quoted in: mechanical horsepower is 745.7 W, but metric horsepower — PS, cv, ch — is 735.5 W, and European engine ratings are usually the latter, a 1.4% difference that quietly walks into converted specifications. Third,  $t$  must be the time over which the work was actually done, not the length of the shift; a hoist that lifts for 20 s and then sits idle for 10 minutes has a duty cycle, and averaging over the whole ten minutes describes the energy consumption honestly but badly understates the motor the job needs.

**Worked example 3.5.** Given  $W = 3000 \text{ J}$ ,  $t = 60 \text{ s}$ ; find  $P$  (power). *3000 J in 60 s  $\rightarrow$  50 W.*

### 3.6 Power from Force and Velocity ( $P = Fv$ )

(Physics 11-15)  $P = Fv$

Where,

$P$  = Power (W)

$F$  = Force (N)

$v$  = Velocity (m/s)

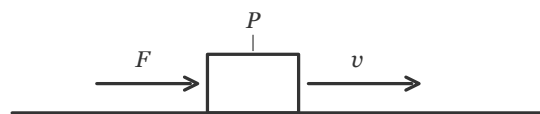


Figure 3.6: power from force and velocity ( $p = fv$ ).

Divide both sides of  $W = Fd$  by time and the  $d/t$  turns into velocity, leaving  $P = Fv$ . It is the same statement as  $P = W/t$ , rewritten for the common case where a steady force is pushing something along at a steady speed — a car holding a cruise, a conveyor dragging material, a tug pulling a barge. The virtue of this form is that it needs no clock and no distance, only what is happening right now.

A car on the highway is fighting drag and rolling resistance. If those total 600 N at 30 m/s — about 108 km/h — the engine must deliver  $P = 600 \times 30 = 18\,000 \text{ W}$ , or 18 kW,

roughly 24 hp, purely to keep the needle where it is. Nothing is accelerating and no height is being gained; that power is going straight into stirring air and warming tyres.

The relation has an unpleasant surprise buried in it for anyone chasing top speed. Aerodynamic drag rises with the *square* of speed, so the power needed to overcome it rises with the *cube*. Doubling highway speed takes roughly eight times the power, which is why an engine of twice the output buys only about a 26% higher top speed, and why fuel

consumption climbs so steeply above about 90 km/h. The rotational version,  $P = \tau\omega$ , is the same equation on a shaft and is what a dyno chart is plotting.

**The conceptual trap is expecting power to feel like force.**

At a fixed power the two trade off exactly: a truck in low gear applies enormous force at a crawl, and the same engine in top gear applies a small force at speed, with the identical power in both cases. That is the whole job of a gearbox. It also means the equation misbehaves at the ends — at  $v$  near zero it would demand infinite force for any finite power, and what actually limits you there is traction and the clutch, not the engine. Two mechanical cautions as well.  $F$  must be the component of force *along* the motion; for a force at an angle, take  $F \cos \theta$  first, exactly as in the work formula. And this is instantaneous power unless both  $F$  and  $v$  hold steady — during acceleration both are changing, and the average power over the run is not  $F_{\text{avg}}v_{\text{avg}}$ .

**Worked example 3.6.** Given  $F = 500 \text{ N}$ ,  $v = 30 \text{ m/s}$ ; find  $P$  (power). *500 N at 30 m/s  $\rightarrow$  15 kW.*

### 3.7 Machine Efficiency

(Physics 11-16) 
$$\eta = \frac{W_{\text{out}}}{W_{\text{in}}}$$

Where,

$\eta$  = Efficiency

$W_{\text{out}}$  = Useful work output (J)

$W_{\text{in}}$  = Work input (J)

No machine returns everything you put in: friction, elastic hysteresis, windage and noise take a cut, so  $\eta = W_{\text{out}}/W_{\text{in}}$  always lands below 1. Feed a hoist 1000 J and get 750 J of lifting done and it is 75% efficient, with 250 J warming the bearings. Enter  $\eta$  as a plain ratio or switch the unit to % — the calculator handles both.

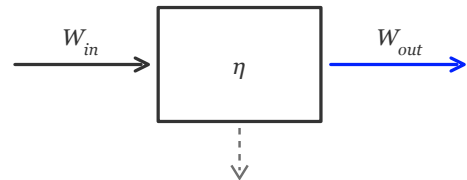


Figure 3.7: machine efficiency.

The historical spread is enormous. Thomas Newcomen's 1712 atmospheric engine converted well under 1% of its coal's energy into work; Watt's separate condenser roughly tripled that; a modern combined-cycle gas turbine reaches about 60%, a large electric motor 95%, and a bicycle drivetrain around 97%. Efficiency chains multiply, which is why a 90%-efficient gearbox behind a 90%-efficient motor delivers 81% overall — and why the honest way to quote a system is end to end, not stage by stage.

**Worked example 3.7.** Given  $W_{\text{out}} = 750 \text{ J}$ ,  $W_{\text{in}} = 1000 \text{ J}$ ; find  $\eta$  (efficiency). *750 J out of 1000 J  $\rightarrow \eta = 0.75$ .*

## Module 3 practice

**25.** Work is a push through a distance: A child pulls a wagon along a straight path with a steady 20 N force, force and motion aligned, doing 300 J of work. *Determine how far the wagon moves.*

**26.** Work is a push through a distance: A tow rope does 750 J of work dragging a sled 15.0 m across level snow, pulling parallel to the ground the whole way. *Determine the tension in the rope.*

**27.** The v-squared surprise: A delivery drone cruising at 4 m/s carries 56 J of kinetic energy. *Determine the drone's mass.*

**28.** The v-squared surprise: A 6 kg remote-controlled car carries 48 J of kinetic energy across the gym floor. *Determine the car's speed.*

**29.** Height is stored energy: A stage crew hoists a 10 kg speaker 10.0 m above the deck and ties it off. *Calculate the gravitational potential energy the speaker gains.*

**30.** Height is stored energy: An elevator counterweight stores 3920 J of gravitational potential energy sitting 8.0 m up its shaft. *Determine the mass of the counterweight.*

**31.** The energy swap: During a stunt rehearsal, a 5 kg sandbag is released from rest 40 m above a crash mat. *Determine the sandbag's stored energy at release, and its speed as it reaches the mat.*

**32.** The energy swap: A 80 kg cliff diver drops from rest and enters the water at 14 m/s. *Determine the diver's kinetic energy at entry, and the height of the takeoff point.*

**33.** How fast is the work done: A dockside winch does 10000 J of work hauling a boat up its ramp in 50.0 s. *Calculate the winch's average power.*

**34.** How fast is the work done: A conveyor drive supplies 2500 W against a steady 500 N of belt load. *Determine the belt's speed.*

**35.** Nothing is perfect: A block-and-tackle delivers 480 J of useful lifting work for every 600 J the crew puts in; the rest leaves as friction, heat and creak. *Calculate the system's efficiency.*

**36.** Nothing is perfect: An electric kettle is 85% efficient at getting energy into the water. One run draws 800 J from the wall. *Determine the useful energy delivered to the water.*

**37.** The Drop Tower: Final page of the paper. A drop-tower ride car of mass 200 kg is winched to the top of a 45 m tower and released, falling freely through the full drop. At the bottom, magnetic brakes cut its speed to half for the creep to the platform, and the winch then hauls it back to the top in 90 s. Take  $g = 10 \text{ m/s}^2$  and work each line — every answer feeds the next. *Determine the energy at the top, the speed at the bottom, the brakes' bill, and the winch's power.*

## MODULE 4

## Waves & sound

Anatomy of a wave · The universal wave equation · The speed of sound · Strings that sing · Pipes and columns · How loud is loud · Moving sources and beats · The Sound Check

### 4.1 Period-Frequency Relation

(Physics 11-17)  $T = \frac{1}{f}$

Where,

$T$  = Period (s)

$f$  = Frequency (Hz)

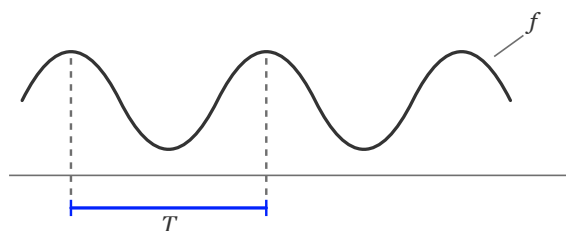


Figure 4.1: period-frequency relation.

Period and frequency are one fact counted in opposite directions. The period  $T$  is seconds per cycle; the frequency  $f$  is cycles per second; and  $T = 1/f$  is not a discovery about nature but unit algebra. If something completes four cycles in a second, each cycle takes a quarter of a second, and no experiment was required to establish that. What earns this relation a page of its own is that almost every oscillation formula you will meet returns one member of the pair while the question in front of you wants the other, so this conversion sits quietly in the middle of nearly every wave calculation.

North American mains alternates at 60 Hz, so one full cycle takes  $1/60 = 16.67$  ms — the interval behind the familiar hum in audio gear. European mains at 50 Hz gives 20 ms. Concert A at 440 Hz gives 2.27 ms per cycle. Run it the other way and a resting heart beating once every 0.8 s is oscillating at 1.25 Hz, which is the same statement as 75 beats per minute.

The unit is younger than the idea. Frequency was written "cycles per second" well into the twentieth century; the International Electrotechnical Commission proposed **hertz**

in 1930 and SI adopted it in 1960, honouring Heinrich Hertz, who between 1886 and 1888 generated and detected radio waves in his Karlsruhe laboratory and so turned Maxwell's equations from mathematics into an observed fact. The hertz is dimensionally just  $\text{s}^{-1}$ , and it is reserved by convention for periodic phenomena — the becquerel is also  $\text{s}^{-1}$  and counts random decays, which is precisely why the two units are kept apart despite being numerically identical. Combined with  $v = f\lambda$ , this page also gives the equally useful  $v = \lambda/T$ .

Three traps. The first is **angular** frequency:  $\omega = 2\pi f$  in radians per second, and the pendulum and spring formulas carry their  $2\pi$  for exactly this reason. Substituting an  $\omega$  where an  $f$  belongs makes the answer wrong by a factor of 6.283, which is large enough to notice and small enough to rationalise. The second is revolutions per minute: 3600 rpm is 60 Hz, not 3600, and the conversion is a division by 60. The third catches people with pendulums — a "seconds pendulum" ticks once per second but has a period of *two* seconds, because a full cycle is out and back. Count a cycle as a return to the starting state moving in the starting direction, and the reciprocal will behave.

**Worked example 4.1.** Given  $f = 60$  Hz; find  $T$  (period). *60 Hz mains  $\rightarrow T = 1/60$  s.*

## 4.2 Wave Speed ( $v = f\lambda$ )

(Physics 11-18)  $v = f\lambda$

Where,

$v$  = Wave speed (m/s)

$f$  = Frequency (Hz)

$\lambda$  = Wavelength (m)

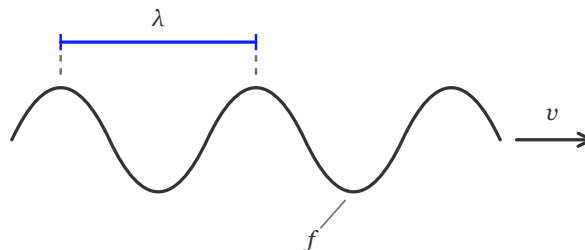


Figure 4.2: wave speed ( $v = f\lambda$ ).

Every traveling wave advances exactly one wavelength during each cycle of its source, and it completes  $f$  cycles every second — so its speed is simply frequency times wavelength. The relation holds for every wave in nature: sound, light, water ripples, seismic tremors. An FM station broadcasting at 100 MHz emits radio waves that travel at the speed of light, about  $3.00 \times 10^8$  m/s, so each wave is roughly 3 m long — which is why FM antennas are built around three-quarters of a metre, a quarter of a wavelength.

The common trap is thinking a higher frequency makes a wave *faster*. It doesn't: speed is set by the medium alone. Sound in room-temperature air moves at about 343 m/s whether it is a 20 Hz bass rumble ( $\lambda \approx 17$  m) or a 20 kHz whistle ( $\lambda \approx 17$  mm). Raise the frequency and the wavelength shrinks in exact proportion, leaving  $v$  untouched.

**Worked example 4.2.** Given  $v = 1234.8$  km/h,  $f = 0.5$  kHz; find  $\lambda$  (wavelength). *Sound at 1234.8 km/h, 0.5 kHz  $\rightarrow \lambda = 0.686$  m.*

## 4.3 Speed of Sound in Air

(Physics 11-19)  $v = 331.3 + 0.606 T_C$

Where,

$v$  = Speed of sound (m/s)

$T$  = Air temperature ( $^{\circ}\text{C}$ )

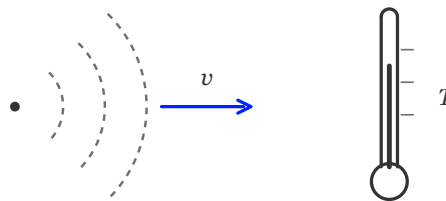


Figure 4.3: speed of sound in air.

Sound travels by molecular collisions, and warmer molecules move faster, passing the disturbance along more quickly. In dry air the speed starts at 331.3 m/s at  $0^{\circ}\text{C}$  and gains about 0.606 m/s per degree: 343 m/s in a  $20^{\circ}\text{C}$  classroom, 349 m/s on a  $30^{\circ}\text{C}$  summer day, but only 325 m/s in  $-10^{\circ}\text{C}$  winter air. The linear formula is an excellent approximation of the true  $\sqrt{T}$  dependence across everyday temperatures.

You use this physics every thunderstorm: light arrives essentially instantly, so counting seconds between flash and rumble and dividing by three gives the storm's distance in kilometres (sound covers about 1 km every 2.9 s at  $20^{\circ}\text{C}$ ). The temperature dependence has audible consequences too — sound bends toward where it travels slower, so on a cold, still morning with warm air above a chilled ground layer, wavefronts curve back down toward the earth and a

distant train sounds uncannily close. Orchestras feel it as well: a flute's pitch rides on  $v$ , drifting sharp as the concert hall warms through the evening.

**Worked example 4.3.** Given  $T = 20\text{ }^{\circ}\text{C}$ ; find  $v$  (speed of sound).  $20\text{ }^{\circ}\text{C air} \rightarrow v = 343.42\text{ m/s}$ .

## 4.4 Fundamental Frequency of a String

(Physics 11-20) 
$$f = \frac{v}{2L}$$

Where,

$f$  = Fundamental frequency (Hz)

$v$  = Wave speed on the string (m/s)

$L$  = String length (m)

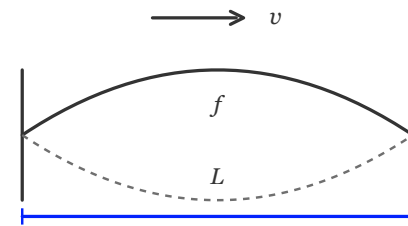


Figure 4.4: fundamental frequency of a string.

A string clamped at both ends cannot move at its ends, and that single boundary condition does all the work. Only standing waves with a node at each clamp can survive; the longest one that fits is a single half-wave, so  $\lambda = 2L$ . Substitute that into the universal wave equation  $v = f\lambda$  and you get  $f = v/2L$ . Nothing else is going on here — this page is the wave equation plus a boundary. The higher modes fit two half-waves, three, four, and so on, giving  $f_n = nv/2L$ : a series of exact integer multiples. That integer relationship is why a string produces a pitched note rather than a noise, and why two strings an octave apart share so many partials that we hear them as almost the same note.

A worked case from a guitar. The standard scale length is 25.5 in, about 648 mm, and the low E string sounds 82.4 Hz. The wave speed along that string is therefore  $v = 2 \times 0.648 \times 82.4 \approx 107\text{ m/s}$ . Press the string at the twelfth fret and the vibrating length halves to 324 mm, which doubles the frequency to 164.8 Hz — one octave up. Every fret position on the neck is this equation solved for  $L$ .

Marin Mersenne published the laws of the vibrating string in *Harmonie universelle* in 1636, decades before Newton: frequency varies inversely with length, inversely with the square root of the string's mass per unit length, and directly with the square root of tension. The last two are bundled into  $v$  here, through  $v = \sqrt{T/\mu}$ , and the page keeps  $v$  as an input rather than burying that relation so you can see which quantity you are actually changing. The same  $f = v/2L$  also gives the fundamental of a pipe open at both ends, where  $v$  is the speed of sound: a 0.65 m open pipe sounds near 264 Hz, roughly middle C.

**The dominant error is entering the speed of sound in air.**

The  $v$  in this equation is the speed of the transverse wave travelling along the string itself, typically 100 to 300 m/s, and it has nothing to do with the 343 m/s at which the resulting sound reaches your ear. The string sets the frequency; the air merely carries it. Two further traps: a pipe *closed* at one end holds a quarter wave, not a half, so its fundamental is  $v/4L$  — an octave lower than an open pipe of the same length, which is why a stopped organ rank saves so much pipe. And  $L$  is the vibrating length between the fixed points, from nut or fret to bridge, not the whole length of the string as it lies on the instrument.

**Worked example 4.4.** Given  $v = 343\text{ m/s}$ ,  $L = 0.5\text{ m}$ ; find  $f$  (fundamental frequency).  $v = 343\text{ m/s}$ ,  $L = 0.5\text{ m} \rightarrow f = 343\text{ Hz}$ .

## 4.5 Wave Speed on a String

(Physics 11-21) 
$$v = \sqrt{\frac{F}{\mu}}$$

Where,

$v$  = Wave speed (m/s)

$F$  = String tension (N)

$\mu$  = Linear mass density (kg/m)

Tension provides the restoring force that snaps a displaced string back; mass per unit length provides the inertia that resists. Their tug-of-war sets the wave speed. A guitar's high E string, with  $\mu \approx 0.4$  g/m under about 70 N of tension, carries waves at  $\sqrt{(70/0.0004)} \approx 418$  m/s — faster than sound in air. Turning a tuning peg raises  $F$ , speeds up the waves, and (since  $f = v/2L$ ) sharpens the pitch.

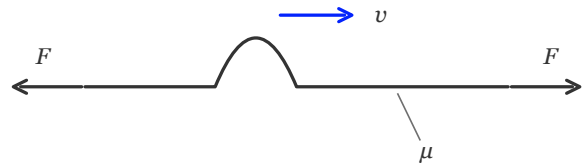


Figure 4.5: wave speed on a string.

The square root explains a luthier's dilemma: to double a string's fundamental you must quadruple its tension, which is why pianos don't just tighten one string type but vary  $\mu$  instead. Bass strings are wrapped in heavy copper windings to raise  $\mu$ , slowing the waves so low notes fit on a playable length — a piano's lowest string can be under more than 1000 N of tension yet still vibrate at a leisurely 27.5 Hz.

**Worked example 4.5.** Given  $F = 100$  N,  $\mu = 0.01$  kg/m; find  $v$  (wave speed).  $100$  N,  $\mu = 0.01$  kg/m  $\rightarrow v = 100$  m/s.

## 4.6 Harmonic Frequencies

(Physics 11-22) 
$$f_n = n f_1$$

Where,

$f_n$  = Harmonic frequency (Hz)

$n$  = Harmonic number

$f_1$  = Fundamental frequency (Hz)

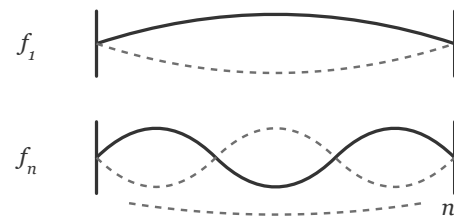


Figure 4.6: harmonic frequencies.

A string or open pipe doesn't resonate at just one frequency but at a whole ladder of them: every whole-number multiple of the fundamental. A guitar string tuned to 110 Hz simultaneously supports 220, 330, 440 Hz and beyond, all ringing at once in proportions that define the instrument's timbre. Touch the string lightly at its midpoint and you silence every odd harmonic, leaving the pure 220 Hz second harmonic — the bell-like "harmonic" trick guitarists use.

The integer ladder is also the foundation of musical harmony. The second harmonic is exactly an octave up; the third is an octave plus a fifth; brass players get every note without moving a valve by "overblowing" up the harmonic series, which is why a bugle can play Taps at all. Remarkably, your brain uses the ladder in reverse: play harmonics 2 through 6 of a 100 Hz tone with the fundamental removed, and you still perceive a 100 Hz pitch — the "missing fundamental" that lets tiny phone speakers imply bass they cannot physically produce.

**Worked example 4.6.** Given  $n = 3$ ,  $f_1 = 110$  Hz; find  $f_n$  (harmonic frequency). *3rd harmonic of 110 Hz → 330 Hz.*

## 4.7 Fundamental of a Closed Pipe

(Physics 11-23) 
$$f = \frac{v}{4L}$$

Where,

$f$  = Fundamental frequency (Hz)

$v$  = Speed of sound (m/s)

$L$  = Pipe length (m)

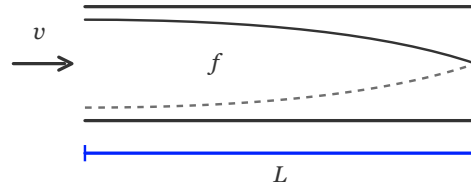


Figure 4.7: fundamental of a closed pipe.

A pipe sealed at one end must have a node (no air motion) at the closed end and an antinode (maximum motion) at the open mouth. The longest wave satisfying both fits just a quarter of a wavelength inside, so the fundamental is  $v/4L$  — an octave lower than an open pipe of the same length. Blow across a 20 cm test tube and you get roughly  $343/(4 \times 0.20) \approx 429$  Hz, close to concert A.

This quarter-wave trick is why a clarinet plays nearly an octave deeper than a flute of the same length: the reed end acts closed while the flute is open at both ends. It also explains the closed pipe's hollow timbre — the boundary conditions only permit odd harmonics ( $f, 3f, 5f, \dots$ ), stripping out every even overtone. Your own ear canal is a closed pipe about 2.5 cm deep, resonating near 3400 Hz, which is precisely where human hearing is most sensitive — and where audiologists find the most noise damage.

**Worked example 4.7.** Given  $v = 343$  m/s,  $L = 0.25$  m; find  $f$  (fundamental frequency). *25 cm closed pipe ( $v = 343$ ) →  $f = 343$  Hz.*

## 4.8 Decibel Sound Level

(Physics 11-24) 
$$\beta = 10 \log_{10} \left( \frac{I}{I_0} \right)$$

Where,

$\beta$  = Sound level (dB)

$I$  = Sound intensity ( $\text{W/m}^2$ )

Because the ear handles intensities spanning twelve orders of magnitude, acousticians compress the scale with a logarithm: every 10 dB step means ten times the intensity. The reference  $I_0 = 10^{-12} \text{ W/m}^2$  is the threshold of hearing, defined as 0 dB. A quiet library at  $10^{-9} \text{ W/m}^2$  works out to  $10 \log_{10}(10^{-9}/10^{-12}) = 30$  dB; a jackhammer at  $10^{-2} \text{ W/m}^2$  hits 100 dB — ten million times the library's intensity, yet only 70 dB more on the scale.

The logarithm produces some counterintuitive arithmetic. Two identical 60 dB violins together do not make 120 dB — doubling the intensity adds only  $10 \log_{10}(2) \approx 3$  dB, for 63 dB total. It takes ten violins to gain a full 10 dB, which the ear perceives as roughly "twice as loud." That is also why workplace noise rules bite so sharply: 85 dB is considered safe for an 8-hour shift, but every 3 dB increase doubles the acoustic energy hitting the ear and halves the safe exposure time.

**Worked example 4.8.** Given  $I = 1 \text{ W/m}^2$ ; find  $\beta$  (sound level). *Threshold of pain:  $1 \text{ W/m}^2 \rightarrow 120 \text{ dB}$ .*

## 4.9 Sound Intensity ( $I = P/A$ )

(Physics 11-25) 
$$I = \frac{P}{A}$$

Where,

$I$  = Sound intensity ( $\text{W/m}^2$ )

$P$  = Acoustic power (W)

$A$  = Area ( $\text{m}^2$ )

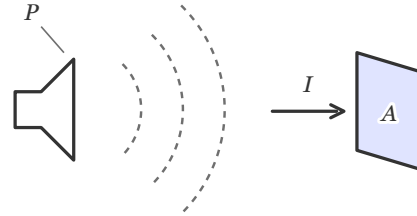


Figure 4.9: sound intensity ( $i = p/a$ ).

Intensity measures how concentrated a sound's energy is: the acoustic power crossing each square meter of surface. A rock band's loudspeaker stack might radiate 100 W of acoustic power; spread that over the 50  $\text{m}^2$  face of a mosh pit and the crowd absorbs  $2 \text{ W/m}^2$  — painfully loud, about 123 dB. The same 100 W spread across a 5000  $\text{m}^2$  festival field ten times farther back drops to a comfortable 0.02  $\text{W/m}^2$ .

What makes hearing remarkable is the range this formula spans. The faintest sound a healthy ear detects carries about  $10^{-12} \text{ W/m}^2$  — a millionth of a millionth of a watt per square meter — while the threshold of pain sits near  $1 \text{ W/m}^2$ , a trillion times greater. A whisper delivering  $10^{-10} \text{ W}$  to your 60  $\text{mm}^2$  eardrum is moving it a distance smaller than the diameter of a hydrogen atom.

**Worked example 4.9.** Given  $P = 100 \text{ W}$ ,  $A = 50 \text{ m}^2$ ; find  $I$  (sound intensity). *100 W over 50  $\text{m}^2 \rightarrow I = 2 \text{ W/m}^2$ .*

## 4.10 Inverse-Square Law for Sound

(Physics 11-26) 
$$I = \frac{P}{4\pi r^2}$$

Where,

$I$  = Sound intensity ( $\text{W/m}^2$ )

$P$  = Acoustic power (W)

$r$  = Distance from source (m)

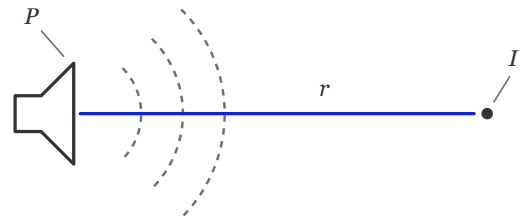


Figure 4.10: inverse-square law for sound.

Sound from a small source spreads out as an ever-growing sphere, and a sphere's surface area grows as  $r^2$ , so the intensity thins out as  $1/r^2$ . Double your distance from a firecracker and the intensity drops to a quarter; step back tenfold and it falls a hundredfold — a 20 dB drop. A 0.5 W siren heard from 10 m delivers  $0.5/(4\pi \times 100) \approx 4 \times 10^{-4} \text{ W/m}^2$ , about 86 dB; from 100 m it is down to 66 dB, no louder than animated conversation.

The law assumes the sound spreads freely in all directions, which is why real environments bend it. A megaphone or a stage horn concentrates power into a cone instead of a sphere, staying loud farther out; a highway heard across open fields behaves more like a line source, fading as  $1/r$  rather than  $1/r^2$ . Concert engineers exploit the pure form in reverse: measure the intensity at a known distance and the formula reveals the source's total acoustic power.

**Worked example 4.10.** Given  $P = 100\text{ W}$ ,  $r = 10\text{ m}$ ; find  $I$  (sound intensity).  $100\text{ W at } 10\text{ m} \rightarrow I = 1/(4\pi r^2) = 1/(4\pi \cdot 10^2) = 1/(400\pi) \approx 0.000796\text{ W/m}^2$ .

### 4.11 Doppler Effect (Approaching Source)

(Physics 11-27) 
$$f' = \frac{fv}{v - v_s}$$

Where,

$f'$  = Observed frequency (Hz)

$f$  = Source frequency (Hz)

$v$  = Speed of sound (m/s)

$v_s$  = Source speed (m/s)

As a source rushes toward you, each successive wavefront is emitted a little closer than the last, squeezing the waves together and raising the pitch you hear. An ambulance siren emitting 700 Hz while driving at 30 m/s through 343 m/s air reaches you at  $700 \times 343/(343 - 30) \approx 767\text{ Hz}$  — about a semitone and a half sharp. The instant it passes, the geometry flips, the waves stretch, and the pitch drops: the classic "nee-naw... nyooow" every child imitates.

**Worked example 4.11.** Given  $f = 700\text{ Hz}$ ,  $v = 340\text{ m/s}$ ,  $v_s = 40\text{ m/s}$ ; find  $f'$  (observed frequency).  $700\text{ Hz siren at } 40\text{ m/s } (v = 340) \rightarrow f' = 793.33\text{ Hz}$ .

### 4.12 Doppler Effect (Approaching Observer)

(Physics 11-28) 
$$f' = \frac{f(v + v_o)}{v}$$

Where,

$f'$  = Observed frequency (Hz)

$f$  = Source frequency (Hz)

$v$  = Speed of sound (m/s)

$v_o$  = Observer speed (m/s)

When you move toward a stationary source, the waves themselves are unchanged — you simply run into them more often. Cycling at 10 m/s toward a 500 Hz factory whistle in 343 m/s air, you intercept  $500 \times (343 + 10)/343 \approx 515\text{ Hz}$ , a shift of 15 Hz. Unlike the moving-source case there is no runaway denominator: even sprinting at the speed of sound

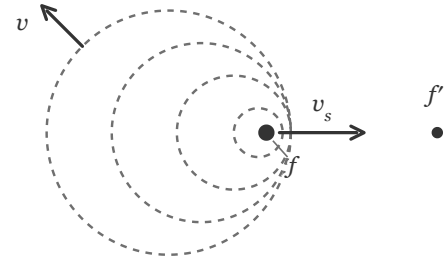


Figure 4.11: doppler effect (approaching source).

The denominator tells a dramatic story of its own. As  $v_s$  approaches the speed of sound, the wavefronts pile up on top of each other and  $f'$  grows without bound — the physical wall of compressed air that early jet pilots called the sound barrier. Cross it and the piled-up fronts trail behind as a shock cone: the sonic boom. Police radar and medical Doppler ultrasound run the same relation in reverse, converting a measured frequency shift into the speed of a car or of blood cells in an artery.

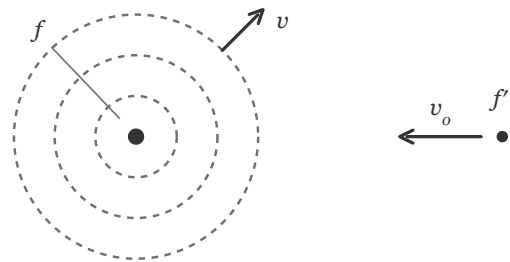


Figure 4.12: doppler effect (approaching observer).

toward the whistle would only double the frequency, because the waves keep their spacing and you just sweep through them faster.

That asymmetry — moving source squeezes the wavelength, moving observer changes only the encounter rate — is a favorite exam trap, and the two formulas give slightly

different answers for the same relative speed. At everyday speeds the difference is tiny: both predict about a 3% shift at 10 m/s. Bats live by this physics, correcting their

echolocation chirps for their own flight speed so the returning echo lands in the narrow frequency band where their ears are sharpest.

**Worked example 4.12.** Given  $f = 500$  Hz,  $v = 343$  m/s,  $v_o = 34.3$  m/s; find  $f'$  (observed frequency). *500 Hz, observer at 34.3 m/s ( $v = 343$ )  $\rightarrow f' = 550$  Hz.*

### 4.13 Beat Frequency

(Physics 11-29)  $f_{\text{beat}} = f_1 - f_2$

Where,

$f_{\text{beat}}$  = Beat frequency (Hz)

$f_1$  = Higher frequency (Hz)

$f_2$  = Lower frequency (Hz)

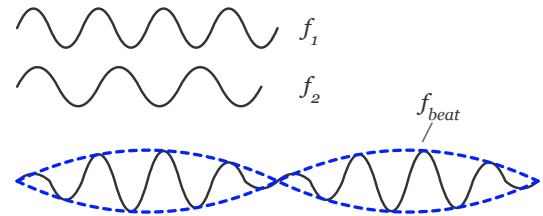


Figure 4.13: beat frequency.

Play two tones that are almost — but not quite — the same, and they drift in and out of step: reinforcing when their crests align, cancelling when they oppose. The result is a single tone that throbs in loudness at exactly the difference of the two frequencies. Strike a 440 Hz tuning fork next to a guitar string vibrating at 437 Hz and you hear three slow pulses per second (by convention we label the higher tone  $f_1$ , so the beat is always a positive number).

Musicians tune by driving this number to zero: as the string approaches the fork's pitch the beats slow — three per second, then one, then a long slow swell — vanishing entirely at unison. Piano tuners go further, deliberately setting certain intervals to beat at prescribed rates to lay an equal-tempered scale by ear alone. The ear stops hearing distinct beats past roughly 15 Hz of separation; push further apart and the throb dissolves into the rough dissonance that music theory has been negotiating for centuries.

**Worked example 4.13.** Given  $f_1 = 440$  Hz,  $f_2 = 437$  Hz; find  $f_{\text{beat}}$  (beat frequency). *440 Hz fork vs 437 Hz string  $\rightarrow 3$  beats/s.*

### Module 4 practice

**38.** Anatomy of a wave: A speaker cone vibrates at 8 Hz. *Determine the period of one cycle.*

**39.** Anatomy of a wave: A lighthouse beam sweeps past every 0.2 s. *Calculate the frequency.*

**40.** The universal wave equation: A wave machine drives waves across a pool at 5 Hz; they travel at 15 m/s. *Determine the wavelength.*

**41.** The universal wave equation: A student shakes one end of a long stretched spring, sending waves of wavelength 4 m along it at 3 Hz. *Calculate the speed of the waves.*

**42.** The speed of sound: A hiker shouts across a canyon on a 15 °C day and hears the echo return 4.0 s later. *Calculate the speed of sound in this air, then the distance to the canyon wall.*

**43.** The speed of sound: Lightning flashes over the lake on a 30 °C evening; the thunder arrives 5.0 s after the flash reaches your eyes. *Calculate the speed of sound, then how far away the strike was.*

**44.** Strings that sing: Waves run at 264.6 m/s along a string whose fundamental is 294 Hz. *Determine the vibrating length of the string.*

**45.** Strings that sing: Waves run at 616 m/s along a string whose fundamental is 440 Hz. *Determine the vibrating length of the string.*

**46.** Pipes and columns: An organ pipe closed at its base has an air column 0.25 m long; the speed of sound in the hall is 344 m/s. *Calculate the fundamental frequency of the pipe.*

**47.** Pipes and columns: In the resonance-tube lab, a tuning fork of 172 Hz is held over a tube; the speed of sound in the room is 344 m/s. The water level is lowered until the first loud resonance is heard. *Determine the length of the air column at that first resonance.*

**48.** How loud is loud: A sound meter at the library reading room reads an intensity of  $1.0 \times 10^{-8} \text{ W/m}^2$ . *Determine the sound level in decibels.*

**49.** How loud is loud: An outdoor siren radiates 80 W of sound power evenly through a surface of area 25 m<sup>2</sup>. *Calculate the sound intensity at that surface.*

**50.** Moving sources and beats: A stunt plane dives toward the airshow crowd at 85 m/s, its engine droning at 450 Hz; a listener stands still beside the track of its approach. The speed of sound that day is 340 m/s. *Calculate the frequency the listener hears.*

**51.** Moving sources and beats: A commuter drives at 34 m/s toward a stationary tornado siren broadcasting 550 Hz. The speed of sound that day is 340 m/s. *Calculate the frequency the moving listener hears.*

**52.** The Sound Check: Sound check in the concert hall, last page of the paper. The hall sits at 20 °C. The reference tone from the mixing desk is 85 Hz; the bass string on stage has a vibrating length of 1.0 m, and waves run along it at 176 m/s. For every line after the first, the paper instructs: take the speed of sound as 340 m/s. Work each line — every answer feeds the next. *Determine the speed of sound, the reference tone's wavelength, the string's fundamental, then the beat.*

**53.** The Sound Check: Bonus mark. The last string of the night is tuned SHARP: sounded against a 384 Hz fork it beats 3 times each second. *Determine the string's frequency.*

## MODULE 5

## Electricity & magnetism

Counting charge · Ohm's law · Series and parallel · Three faces of power · The hydro bill · Magnets at work · The Breaker Panel

### 5.1 Electric Charge ( $Q = It$ )

(Physics 11-30)  $Q = It$

Where,

$Q$  = Charge (C)

$I$  = Current (A)

$t$  = Time (s)

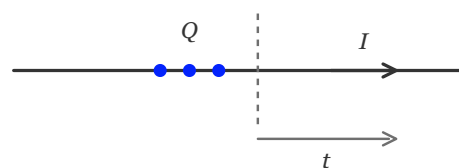


Figure 5.1: electric charge ( $q = it$ ).

Current is not a thing that flows. It is a *rate* — the amount of charge passing a chosen cross-section of the conductor each second — and one ampere means one coulomb per second. Once that is clear,  $Q = It$  needs no proof, because it is the definition read backwards: if charge crosses at a steady rate, the total that crossed is the rate multiplied by how long it kept up. The only condition the equation imposes is the word **steady**. A current that varies has to be integrated,  $Q = \int I dt$ , and this page is the special case where the integral collapses to a rectangle.

A 2 A charger running for one hour moves  $2 \times 3600 = 7200$  C. Battery ratings are the same arithmetic wearing different units: a phone cell marked 3000 mAh holds 3 Ah, and  $3 \times 3600 = 10\,800$  C of deliverable charge. Divide by the elementary charge,  $1.602 \times 10^{-19}$  C, and that is about  $6.7 \times 10^{22}$  electrons — a number that only sounds absurd until you remember a gram of copper contains ten times as many free ones already sitting in the metal, drifting at well under a millimetre per second.

Since the 2019 redefinition of the SI, this relation is closer to the foundation than it used to be. The ampere is now fixed by declaring the elementary charge to be exactly  $1.602176634 \times 10^{-19}$  C, which makes the coulomb a *count* of charges and the ampere a count per second.

Michael Faraday got there experimentally in the 1830s: his laws of electrolysis measure the charge needed to plate out a mole of a substance, and that constant — 96 485 C per mole — is nothing but  $Q = It$  run on a plating tank. Electroplating, anodising and battery capacity testing all still bill in ampere-hours for exactly this reason.

**Two errors are worth naming, and one convention deserves an apology.** The first error is treating milliamp-hours as energy. They are charge; a 3000 mAh cell at 3.7 V holds  $3 \times 3.7 = 11.1$  Wh, and the same 3000 mAh at 1.2 V holds a third of that, so comparing two batteries by mAh alone tells you very little. The second is applying the equation to a current that is not constant — a motor's inrush, a switching supply's chopped input, or anything on AC, where over a full cycle the net charge transferred is zero even though the current is real all along. As for the convention: current is drawn flowing from plus to minus, while in a metal the electrons actually travel the other way. Benjamin Franklin guessed the sign in the 1750s, a century before anyone knew a charge carrier existed, and he guessed wrong. Nothing in the physics breaks — a deficit of negatives moving left is indistinguishable from positives moving right — but it is a historical accident, not a discovery, and it is worth knowing that it is one.

**Worked example 5.1.** Given  $I = 2 \text{ A}$ ,  $t = 30 \text{ s}$ ; find  $Q$  (charge).  $2 \text{ A for } 30 \text{ s} \rightarrow 60 \text{ C}$ .

## 5.2 Ohm's Law

(Physics 11-31)  $V = IR$

Where,

$V$  = Voltage (V)

$I$  = Current (A)

$R$  = Resistance ( $\Omega$ )

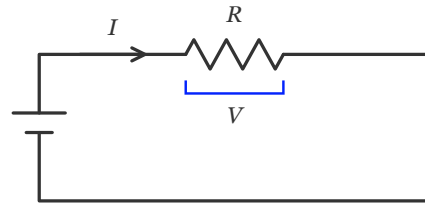


Figure 5.2: ohm's law.

Georg Ohm published this relation in 1827 after painstaking experiments with wires of different lengths and thicknesses — and was initially ridiculed for reducing electricity to arithmetic. The idea is simple: voltage is the electrical *push*, resistance is the *opposition*, and current is what results. Double the push and you double the flow; double the opposition and you halve it. A 12 V car battery connected across a  $6 \Omega$  lamp drives  $12/6 = 2 \text{ A}$  through it; swap in a  $3 \Omega$  lamp and the current doubles to 4 A.

The law holds for **ohmic** conductors at constant temperature — metals, resistors, most wiring. Components like diodes, filament bulbs, and thermistors bend the rule because their resistance shifts as they heat up or as voltage changes. The classic V–I–R triangle mnemonic works because every rearrangement here is a single multiplication or division.

**Worked example 5.2.** Given  $I = 2 \text{ A}$ ,  $R = 6 \Omega$ ; find  $V$  (voltage).  $2 \text{ A through } 6 \Omega \rightarrow 12 \text{ V}$ .

## 5.3 Two Resistors in Series

(Physics 11-32)  $R_t = R_1 + R_2$

Where,

$R_t$  = Total resistance ( $\Omega$ )

$R_1$  = Resistance 1 ( $\Omega$ )

$R_2$  = Resistance 2 ( $\Omega$ )

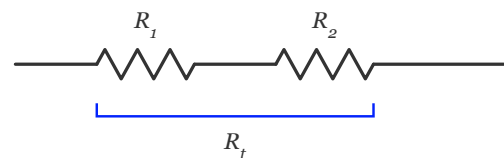


Figure 5.3: two resistors in series.

Two resistors wired end to end sit on one path, and that single fact settles everything else. Charge has nowhere to go but forward, so the **same current passes through both** — it is not divided between them and it is not shared out according to size. Each resistor then takes its own voltage drop,  $V_1 = IR_1$  and  $V_2 = IR_2$ , and the two drops must add up to whatever the supply provides. Divide that sum by

the current they have in common and the resistances add:  $R_t = R_1 + R_2$ . This is not a rule to memorise. It is Ohm's law applied twice in a circuit that has only one loop.

Put a  $47 \Omega$  resistor in series with a  $220 \Omega$  resistor across a 12 V supply. The total is  $267 \Omega$ , so the current is  $12/267 = 45 \text{ mA}$ , and it is 45 mA at every point in the loop — before the first resistor, between them, and after the second. The  $47 \Omega$  part drops  $0.045 \times 47 = 2.1 \text{ V}$ ; the  $220 \Omega$  part drops

$0.045 \times 220 = 9.9 \text{ V}$ ; and  $2.1 + 9.9$  gives back the  $12 \text{ V}$  we started with. Adding the drops as a check costs nothing and catches most arithmetic errors on the spot.

The bookkeeping behind that check is Kirchhoff's voltage law, published by Gustav Kirchhoff in 1845 while he was still a student: go once around any closed loop and the voltage rises equal the voltage falls, because the loop returns you to the potential you started at. Every series result descends from it. The voltage divider is the same equation rearranged — each resistor claims the fraction  $R_1/(R_t)$  of the supply — and that is how a potentiometer, a thermistor bridge and a sensor's biasing network all work. Solving this page backwards for one resistor is subtraction,  $R_1 = R_t - R_2$ , which is why the total must exceed the branch you already know.

**The mistakes cluster in three places.** The first is reaching for the wrong combination rule: series resistors add, parallel resistors combine as product over sum, and *capacitors do exactly the reverse* — series capacitors are the reciprocal case. Whenever you find yourself using product-over-sum on a series string, stop and ask which component you are holding. The second is assuming the larger resistor gets the larger current; it gets the larger *voltage drop* at the same

current, and confusing those two makes a mess of any divider. The third only appears on AC: a coil or a capacitor in series with a resistor cannot be added arithmetically to it. Reactance is  $90^\circ$  out of phase with resistance, so a  $30 \Omega$  resistor in series with  $40 \Omega$  of reactance presents  $\sqrt{30^2 + 40^2} = 50 \Omega$ , not  $70$ . This page adds resistances, and resistances are what it will add — the quadrature sum belongs on the impedance pages.

**Worked example 5.3.** Given  $R_1 = 220 \Omega$ ,  $R_2 = 330 \Omega$ ; find  $R_t$  (total resistance).  $220 \Omega + 330 \Omega$  in series  $\rightarrow 550 \Omega$ .

## 5.4 Two Resistors in Parallel

(Physics 11-33) 
$$R_t = \frac{R_1 R_2}{R_1 + R_2}$$

Where,

$R_t$  = Total resistance ( $\Omega$ )

$R_1$  = Resistance 1 ( $\Omega$ )

$R_2$  = Resistance 2 ( $\Omega$ )

Wired side by side, two resistors give the current two paths at once, so more current flows for the same voltage and the combination resists *less* than either branch alone. The tidy product-over-sum form is just  $1/R_t = 1/R_1 + 1/R_2$  rearranged — conductances, not resistances, are what add

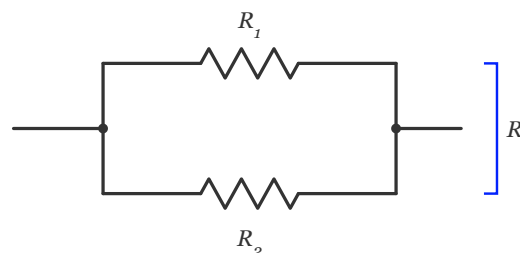


Figure 5.4: two resistors in parallel.

in parallel. Worked example:  $100 \Omega$  in parallel with  $25 \Omega$  gives  $(100 \times 25)/(100 + 25) = 2500/125 = 20 \Omega$ , comfortably below the smaller branch.

Two handy special cases: equal resistors in parallel halve (two  $100\ \Omega$  resistors make  $50\ \Omega$ ), and a much smaller resistor dominates —  $10\ \Omega$  in parallel with  $10\ \text{k}\Omega$  is essentially  $10\ \Omega$ . Household outlets are wired in parallel so

every appliance sees full mains voltage. When solving for a branch, the other resistance must exceed the total, since the total is always the smallest value in the circuit.

**Worked example 5.4.** Given  $R_1 = 4\ \Omega$ ,  $R_2 = 12\ \Omega$ ; find  $R_t$  (total resistance).  $4\ \Omega \parallel 12\ \Omega \rightarrow 3\ \Omega$ .

## 5.5 Electrical Power ( $P = VI$ )

(Physics 11-34)  $P = VI$

Where,

$P$  = Power (W)

$V$  = Voltage (V)

$I$  = Current (A)

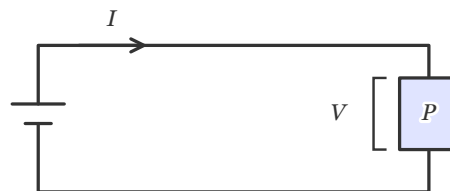


Figure 5.5: electrical power ( $p = vi$ ).

This formula falls straight out of the definitions. A volt is a joule per coulomb — the energy each unit of charge carries — and an ampere is a coulomb per second — how many units of charge arrive each second. Multiply them and the coulombs cancel, leaving joules per second: watts. A  $1500\ \text{W}$  space heater on a  $120\ \text{V}$  household circuit draws  $1500/120 = 12.5\ \text{A}$ , which is why it crowds a standard  $15\ \text{A}$  breaker and shouldn't share the circuit with much else.

Unlike the resistor-specific forms  $P = I^2R$  and  $P = V^2/R$ , this version works for **any** component — ohmic or not — including motors, LEDs, and batteries, because it comes from the definitions of voltage and current rather than from Ohm's law. Electric utilities meter exactly this product, accumulated over time, when they bill you for energy.

**Worked example 5.5.** Given  $V = 120\ \text{V}$ ,  $I = 0.5\ \text{A}$ ; find  $P$  (power).  $120\ \text{V at } 0.5\ \text{A} \rightarrow 60\ \text{W}$ .

## 5.6 Electrical Power ( $P = I^2R$ )

(Physics 11-35)  $P = I^2R$

Where,

$P$  = Power (W)

$I$  = Current (A)

$R$  = Resistance ( $\Omega$ )

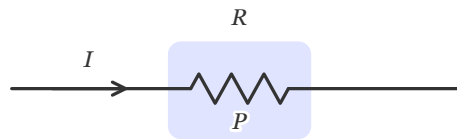


Figure 5.6: electrical power ( $p = i^2r$ ).

This is the heat form of electrical power, and the square is the whole point of it. Start from  $P = VI$ , substitute Ohm's law for the voltage across the resistance,  $V = IR$ , and you get  $P = I^2R$ . Because the current appears twice, the heat does not track the load — it tracks the *square* of the load.

Double the current and a conductor dissipates four times the heat; triple it and nine times. Nothing else in ordinary wiring punishes a modest overload so hard, and it is the reason a circuit that runs warm at rated current runs dangerously hot at 150% of it.

Take a 30 m branch circuit run in 12 AWG copper. The conductor is about  $3.31 \text{ mm}^2$ , copper's resistivity is  $1.68 \times 10^{-8} \Omega \cdot \text{m}$ , so each metre is roughly  $5.1 \text{ m}\Omega$  — and the current has to go out and come back, so the loop is 60 m and about  $0.30 \Omega$ . At 15 A the copper dissipates  $15^2 \times 0.30 = 68 \text{ W}$ , spread along the run, and drops  $15 \times 0.30 = 4.6 \text{ V}$  of the supply before it ever reaches the load. Drop the current to 5 A and the loss falls to 7.5 W, not to a third: that is the square doing its work.

James Joule established the law in 1841 by immersing coils in water and measuring the temperature rise, which was also his route to the mechanical equivalent of heat and thence to the first law of thermodynamics. The same relation is the reason the grid transmits at hundreds of kilovolts. Delivered power is  $VI$ , so raising the voltage a hundredfold cuts the current a hundredfold for the same power, and cuts the line loss by ten thousand. Every transformer between a generating station and a house exists to move a fixed quantity of watts into a lower current, purely so that this equation returns a smaller number.

**Worked example 5.6.** Given  $I = 3 \text{ A}$ ,  $R = 10 \Omega$ ; find  $P$  (power). *3 A through  $10 \Omega \rightarrow 90 \text{ W}$ .*

## 5.7 Electrical Power ( $P = V^2/R$ )

(Physics 11-36) 
$$P = \frac{V^2}{R}$$

Where,

$P$  = Power (W)

$V$  = Voltage (V)

$R$  = Resistance ( $\Omega$ )

This is the same power as  $P = VI$ , written for the situation you actually meet at a wall outlet: the voltage is fixed and the resistance is what you choose. Substitute  $I = V/R$  into  $P = VI$  and the current disappears, leaving  $P = V^2/R$ . Read it carefully, because the two variables behave in opposite directions. Power rises with the *square* of the voltage but falls *inversely* with resistance — so halving the resistance doubles the power, while doubling the voltage quadruples it.

**Three traps, in rising order of consequence.** First,  $R$  is the resistance *of the thing dissipating the heat*, and  $I$  is the current through that same thing — mixing the conductor's resistance with the load's current is fine only because they are in series, and on a branched circuit it is not fine at all. Second, on AC the current must be RMS. A peak reading gives twice the power for a sine wave, and using it is one of the most common ways to double an answer without noticing. Third, and the one that bites in the field: this equation uses *resistance*, and on an AC circuit the opposition to current is impedance. A long run of steel-armoured cable, a coil, or a motor feeder has reactance as well as resistance, and the voltage drop computed from DC resistance alone will understate the real drop. The heat, though, still comes only from the resistive part — reactance stores energy and hands it back, so it moves voltage around without ever warming the copper.

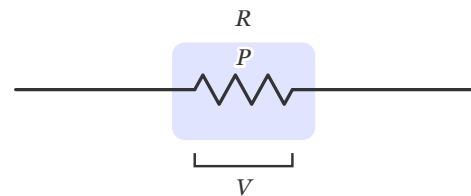


Figure 5.7: electrical power ( $p = v^2/r$ ).

A 1500 W kettle on a 120 V supply must therefore be built with  $R = V^2/P = 14\,400/1500 = 9.6 \Omega$  of element. The same element plugged into 240 V would try to deliver  $57\,600/9.6 = 6000 \text{ W}$  — four times its rating — which is why travel appliances fail spectacularly rather than gradually. Run the arithmetic the other way for a 1000 W element on 230 V mains and you need about  $53 \Omega$ . Notice that the element's resistance is a design choice made to hit

a wattage at one particular voltage; the wattage on the label is not a property of the element, it is a property of the element *and* the supply it was designed for.

The three power forms,  $P = VI$ ,  $P = I^2R$  and  $P = V^2/R$  — are one equation seen from three sides, and picking the right one is mostly about which quantity is being held constant. Devices in *parallel* across a fixed supply are the  $V^2/R$  case: every extra appliance you plug in adds power, because it adds a path, and lowering the effective resistance raises the total draw. Devices in *series* carrying a common current are the  $I^2R$  case. Only  $P = VI$  makes no assumption at all, which is why it is the one that still works for motors, LEDs and batteries where Ohm's law does not.

**Where this goes wrong.** The resistance to use is the resistance *at operating temperature*, and for anything that glows that is not what an ohmmeter reads on the bench. A tungsten filament's resistance climbs by roughly a factor of fifteen between room temperature and incandescence, so a 60 W lamp measuring 20  $\Omega$  cold does not draw 720 W — it draws a large inrush for a few milliseconds and then settles near 240  $\Omega$ . The second error is the mains one: use RMS voltage, always. Plugging the 170 V peak of a 120 V circuit into this formula doubles the answer. The third is applying it to a load that is not resistive. A motor at 120 V drawing 5 A

is not a 24  $\Omega$  resistor; most of that opposition is reactance, which stores and returns energy rather than turning it into heat, and  $V^2/R$  with an impedance in the denominator will overstate the watts by exactly the power factor.

**Worked example 5.7.** Given  $V = 120$  V,  $R = 240$   $\Omega$ ; find  $P$  (power). *120 V across 240  $\Omega \rightarrow 60$  W.*

## 5.8 Electrical Energy ( $E = Pt$ )

(Physics 11-37)  $E = Pt$

Where,

$E$  = Energy (J)

$P$  = Power (W)

$t$  = Time (s)

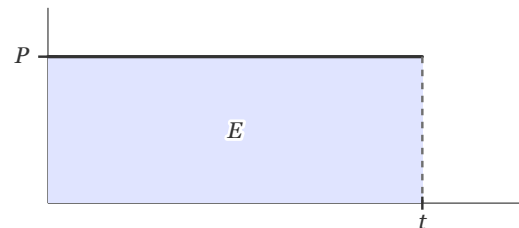


Figure 5.8: electrical energy ( $e = pt$ ).

Power is the rate at which energy is delivered, so energy is power kept up for a while. That is all  $E = Pt$  says, and like every rate-times-time relation it is true by definition rather than by discovery — a watt *is* a joule per second, so watts multiplied by seconds give joules back. The distinction it enforces is the one people most often lose: **power is not**

**energy.** A 2000 W heater is not consuming 2000 of anything; it is consuming at a rate of 2000 joules every second, and what it costs depends entirely on how long you leave it on.

Utilities meter in kilowatt-hours because the joule is inconveniently small: 1 kWh is 1000 W sustained for 3600 s, or exactly  $3.6 \times 10^6$  J. A 1500 W baseboard heater running six hours a day for a thirty-day month uses  $1.5 \times 6 \times 30 = 270$  kWh; at ten cents a kilowatt-hour that is \$27 on the bill. The same 270 kWh would run a 15 W LED lamp continuously for about two years. Energy comparisons like that are the only honest way to judge where a bill actually goes, and they almost always show that the heating and hot water dwarf everything with a screen on it.

The kilowatt-hour is a compound unit of the sort engineers usually avoid, and it survives because it matches how people buy electricity: a rate you can read off a nameplate multiplied by hours you can read off a clock. Watt-hours, ampere-hours and joules all measure the same physical stock of energy in different currencies:  $1 \text{ Wh} = 3600 \text{ J}$ , and an ampere-hour becomes watt-hours only after you

multiply by the voltage. On the site's other pages this same relation appears as work over time in mechanics; there is no separate electrical version of it, only a separate unit.

**The assumption doing the quiet work here is constant power, and most real loads are not.** A thermostatted heater is either fully on or fully off, so its *average* power over an hour is the rated power times its duty cycle — a 1500 W baseboard cycling a third of the time is a 500 W load as far as the meter is concerned, and using the nameplate figure triples the estimate. A refrigerator, a well pump and a furnace blower all behave the same way. The other trap is a billing one worth knowing if you read a commercial invoice: those bills carry both an energy charge in kilowatt-hours and a *demand* charge in kilowatts, set by the highest fifteen-minute average draw in the period. The demand charge is a power charge, this equation does not produce it, and no amount of shortening run times will reduce it — only flattening the peak will.

**Worked example 5.8.** Given  $P = 60 \text{ W}$ ,  $t = 120 \text{ s}$ ; find  $E$  (energy).  $60 \text{ W for } 120 \text{ s} \rightarrow 7200 \text{ J}$ .

## 5.9 Magnetic Force on a Current-Carrying Wire

(Physics 11-38)  $F = BIL \sin \theta$

Where,

$F$  = Magnetic force (N)

$B$  = Magnetic flux density (T)

$I$  = Current (A)

$L$  = Wire length in field (m)

$\theta$  = Angle between wire and  $B$  ( $^\circ$ )

This is the force on one moving charge,  $F = qvB \sin \theta$ , added up over all the charges in a length of wire. A current  $I$  means charge crossing at  $I$  coulombs per second, so a length  $L$  of conductor holds a quantity of moving charge whose product with its drift speed is exactly  $IL$  — the individually feeble pushes on perhaps  $10^{22}$  slowly drifting electrons, collected by the metal lattice and delivered to the wire as a whole. That is why  $F = BIL \sin \theta$  contains no reference to how many carriers there are or how fast they move: those two factors always multiply out to the current.

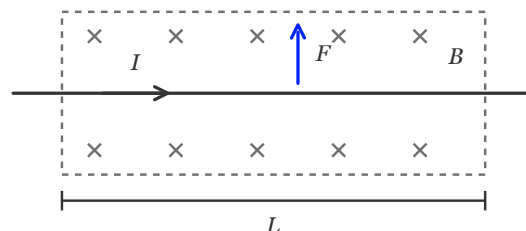


Figure 5.9: magnetic force on a current-carrying wire.

The sine handles orientation, peaking when the wire lies across the field and vanishing when it lies along it, since a charge coasting parallel to a field feels nothing.

A 0.25 m length of wire carrying 8 A across a 0.4 T field at right angles feels  $F = 0.4 \times 8 \times 0.25 = 0.8 \text{ N}$  — about the weight of a coffee mug, from a single conductor. Multiply by a few hundred turns in an armature and you have the torque of a real motor. Tilt the same wire to  $30^\circ$  from the field and the force drops to  $0.8 \sin 30^\circ = 0.4 \text{ N}$ , half. The direction is perpendicular to both the wire and the field, given by the right-hand rule, and this sideways push is what motor designers arrange to be a torque.

Faraday demonstrated the effect in 1821 with a wire free to rotate around a magnet dipped in mercury — the first electric motor, built to settle an argument about whether electromagnetism could produce continuous motion. Every motor and every loudspeaker since is the same experiment industrialised, the cone driven by a coil of wire hanging in a permanent magnet's gap with the audio signal as  $I$ . Note that this relation and the motional-EMF page are two faces of one thing: push current through a wire in a field and it moves, move a wire in a field and current appears, and a motor and a generator are the same machine run in opposite directions.

**Three cautions.** The angle  $\theta$  is measured between the wire and the field, and it cannot be solved for on this page — arcsine cannot tell  $\theta$  from its supplement  $180^\circ - \theta$ , so the calculator returns  $F$ ,  $B$ ,  $I$  or  $L$  but never the angle.  $L$  is the length of conductor *actually inside the field*, not the length of the wire; a metre of lead-in outside the magnet gap contributes nothing. And a note on the right-hand rule: it works with **conventional current**, drawn flowing from

positive to negative, while the electrons in the copper are travelling the other way. Benjamin Franklin fixed that sign a century before the electron was found, and he fixed it backwards. Both descriptions give the same force in the same direction — negative charge moving left is the same current as positive charge moving right — but if you switch to reasoning about electrons you must switch hands too, and mixing the two is the surest way to get a motor turning the wrong way on paper.

**Worked example 5.9.** Given  $B = 0.5 \text{ T}$ ,  $I = 10 \text{ A}$ ,  $L = 2 \text{ m}$ ,  $\theta = 90^\circ$ ; find  $F$  (magnetic force). *10 A in 2 m of wire across 0.5 T  $\rightarrow$  10 N.*

## 5.10 Transformer Voltage Ratio

(Physics 11-39) 
$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

Where,

$V_p$  = Primary voltage (V)

$V_s$  = Secondary voltage (V)

$N_p$  = Primary turns

$N_s$  = Secondary turns

A transformer is two coils sharing one magnetic circuit, and the turns ratio follows from Faraday's law applied to each of them. The alternating current in the primary drives an alternating flux around the iron core; that same flux threads the secondary, because the core is there precisely to make sure it does. Each turn of wire, primary or secondary, has the identical  $\Delta\Phi/\Delta t$  passing through it, and therefore develops the identical volts per turn. Ten times the turns

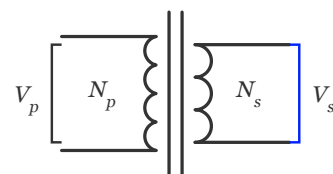


Figure 5.10: transformer voltage ratio.

intercepting the same changing flux means ten times the induced voltage — hence  $V_s/V_p = N_s/N_p$ . Nothing about the wire gauge, the core size or the load enters into it.

A doorbell transformer stepping 120 V down to 16 V has a turns ratio of 7.5 to 1: a 900-turn primary against a 120-turn secondary. Because an ideal transformer neither creates nor destroys power,  $V_p I_p = V_s I_s$ , and the current ratio inverts — that 16 V secondary supplying 1 A draws only 0.13 A from

the 120 V side. Impedance transforms as the *square* of the turns ratio,  $Z_p/Z_s = (N_p/N_s)^2$ , which is the whole reason for the output transformer in a valve amplifier and for matching transformers in radio work.

Michael Faraday wound the first one in 1831 on an iron ring, and the arrangement that carries the modern name was developed by Ottó Bláthy, Miksa Déri and Károly Zipernowsky in Budapest in 1885. It is the reason alternating current won the arguments of the 1890s: no comparably simple device changes DC voltage, and without cheap voltage changing you cannot transmit at high voltage and consume at low. **A transformer works only on changing flux.** Connect a steady DC supply and the induced secondary voltage is zero, while the primary — with only its winding resistance to limit current — draws whatever the supply will give and burns.

**The trap that catches people in the field is that this equation is the ideal, and the ideal is a no-load figure.**

Measure the secondary of a transformer with nothing connected and you will get close to the turns ratio. Load it and the voltage sags, because the winding resistance and the leakage reactance drop voltage inside the transformer itself — a small unit may deliver 10% less than its nameplate at full load, which is exactly why nameplates quote a rated output at a rated current rather than a bare ratio. This is the transformer's version of a point worth stating plainly: an induced EMF is not the same thing as the terminal voltage you can measure once current flows, any more than a battery's EMF equals its terminal voltage under load. Two further cautions: the turns ratio says nothing about isolation or safety, since an autotransformer shares a winding and offers none; and stepping voltage down steps current *up*, so a secondary short is a far more violent event than the primary's fuse rating suggests.

**Worked example 5.10.** Given  $V_p = 120\text{ V}$ ,  $N_p = 500$ ,  $N_s = 25$ ; find  $V_s$  (secondary voltage). *120 V, 500:25 turns  $\rightarrow$  6 V secondary.*

## Module 5 practice

**54.** Counting charge: Over a timed run of 10 s, a charge of 30 C flows through a circuit at a steady rate. *Determine the current in the circuit.*

**55.** Counting charge: A charger supplies a steady 2 A, and the battery needs 120 C to top up. *Determine how long the charge takes to arrive.*

**56.** Ohm's law: A heating element with a resistance of 25  $\Omega$  carries a steady current of 5 A. *Calculate the voltage across the element.*

**57.** Ohm's law: A 10  $\Omega$  resistor is connected across a 60 V supply. *Determine the current through the resistor.*

**58.** Series and parallel: Two resistors, 10  $\Omega$  and 30  $\Omega$ , are wired in series — end to end, one single path — across a 120 V supply. *Determine the current the ammeter reads.*

**59.** Series and parallel: Two resistors, 6  $\Omega$  and 30  $\Omega$ , are wired in parallel — side by side, sharing both ends. *Determine the combined resistance of the pair.*

**60.** Three faces of power: An electric kettle rated 2100 W plugs into the 120 V line. *Determine the current it draws.*

**61.** Three faces of power: A heating element dissipates 40 W while carrying a current of 2 A. *Determine its resistance.*

**62.** The hydro bill: A 1250 W space heater runs for 3.0 h through the off-peak evening window, when electricity costs 12 ¢/kWh. *Determine what the evening costs.*

**63.** The hydro bill: A workshop heater on a 120 V outlet draws 6 A, and runs for 10.0 h through the evening. *Determine the energy the evening costs, in kilowatt-hours.*

**64.** Magnets at work: A wire segment 1.5 m long sits square across a 0.2 T field. The motor design calls for a force of 4.5 N on that segment. *Determine the current required.*

**65.** Magnets at work: A wire segment 0.5 m long sits square across a 0.2 T field. The motor design calls for a force of 0.8 N on that segment. *Determine the current required.*

**66.** The Breaker Panel: Last page of the paper. Two heating elements,  $10\ \Omega$  and  $20\ \Omega$ , are wired in series on a  $120\ \text{V}$  circuit protected by a  $15\ \text{A}$  breaker. Work each line — every answer feeds the next. *Determine the resistance, the current, the power, the evening's energy — then give the breaker's verdict.*

**67.** The Breaker Panel: Bonus mark, worked backwards: a  $20\ \text{A}$  breaker lets its  $120\ \text{V}$  circuit draw right up to the rating and not one ampere more. *Determine the largest appliance power the circuit can carry without tripping.*

## MODULE 6

# Heat

Heat is not temperature ·  $Q = mc\Delta T$ , asked every way · Changing state · Warm it, then boil it · Heat on the move · The stretch of summer · The Kettle

## 6.1 Sensible Heat ( $Q = mc\Delta T$ )

(Physics 11-40)  $Q = mc\Delta T$

Where,

$Q$  = Heat energy (J)

$m$  = Mass (kg)

$c_p$  = Specific heat capacity ( $\text{J}/(\text{kg}\cdot\text{K})$ )

$\Delta T$  = Temperature change ( $^\circ\text{C}$ )

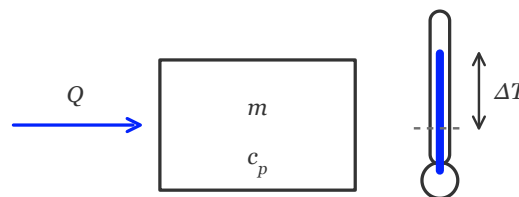


Figure 6.1: sensible heat ( $q = mc\delta t$ ).

Sensible heat is the energy that changes a substance's temperature without changing its phase. The specific heat capacity  $c$  is the price of each degree: how many joules one kilogram demands per kelvin of warming. Water's is famously steep at about  $4186\ \text{J}/(\text{kg}\cdot\text{K})$ , which is why oceans moderate coastal climates and why a kettle takes its time. Heating  $1.5\ \text{kg}$  of water from  $15\ ^\circ\text{C}$  to  $95\ ^\circ\text{C}$  costs  $Q = 1.5 \times 4186 \times 80 \approx 502\ \text{kJ}$ .

The concept dates to Joseph Black's calorimetry experiments in 1760s Glasgow, which first pried apart the ideas of temperature and heat. Note that  $\Delta T$  is a temperature *difference*, so a change of  $80\ ^\circ\text{C}$  equals a change of  $80\ \text{K}$  exactly — Fahrenheit differences convert by scale alone, with no offset. The formula holds as long as  $c$  stays roughly constant over the range and nothing melts or boils along the way.

**Worked example 6.1.** Given  $m = 2\ \text{kg}$ ,  $c_p = 4186\ \text{J}/(\text{kg}\cdot\text{K})$ ,  $\Delta T = 30\ ^\circ\text{C}$ ; find  $Q$  (heat energy). *2 kg water,  $c = 4186$ ,  $\Delta T = 30\ ^\circ\text{C} \rightarrow 251160\ \text{J}$ .*

## 6.2 Latent Heat

(Physics 11-41)  $Q = mL$

Where,

$Q$  = Heat absorbed or released (J)

$m$  = Mass changing phase (kg)

$L$  = Specific latent heat (J/kg)

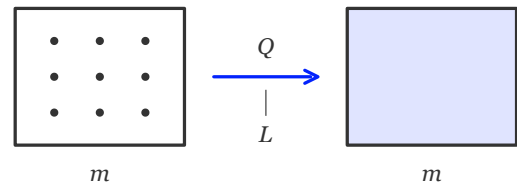


Figure 6.2: latent heat.

Latent heat is the energy a phase change absorbs or releases while the temperature holds still. Melting 1 kg of ice at 0 °C soaks up 334 kJ — enough to heat that same water from 0 °C to 80 °C — yet the thermometer never moves until the last crystal is gone. Boiling is costlier still: vaporizing a kilogram of water takes about 2256 kJ, more than five times the energy needed to warm it from ice-cold to boiling.

Joseph Black coined the term in 1762 — *latent* means hidden, because the heat disappears into the phase change instead of the temperature reading. The physics runs everyday life: sweat cools you as it evaporates, steam scalds far worse than boiling water because it dumps its latent heat

on condensing against skin, and every refrigerator moves heat by evaporating and condensing a working fluid in an endless loop.

One quantity, three notations, depending on whose book you are holding. Physics writes it  $L$ , as here, and splits it into  $L_f$  for fusion and  $L_v$  for vaporization. Engineering and every steam table print it  $h_{fg}$ , where  $f$  is saturated fluid and  $g$  is saturated gas — so  $h_{fg} = h_g - h_f$  is the gap between the two columns, which is exactly why it shrinks to nothing at the critical point where those columns meet. Chemistry writes it as an enthalpy of vaporization per MOLE rather than per kilogram, so its numbers look nothing like these until you divide by the molar mass. Same energy, three addresses.

**Worked example 6.2.** Given  $m = 1$  kg,  $L = 334$  kJ/kg; find  $Q$  (heat absorbed or released). *Melting 1 kg ice at 334 kJ/kg  $\rightarrow$  334000 J.*

## 6.3 Heat Conduction Rate

(Physics 11-42)  $P = \frac{kA\Delta T}{d}$

Where,

$P$  = Heat flow rate (W)

$k$  = Thermal conductivity (W/(m·K))

$A$  = Cross-sectional area (m<sup>2</sup>)

$\Delta T$  = Temperature difference (C°)

$d$  = Thickness (mm)

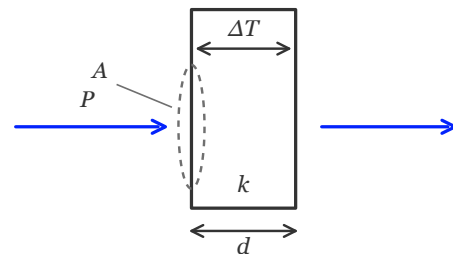


Figure 6.3: heat conduction rate.

Fourier's law says heat flows down a temperature gradient at a rate proportional to how steep that gradient is. Written for a flat slab, the gradient is  $\Delta T/d$  and the flow is  $P = kA\Delta T/d$ . Each term earns its place: more area gives the heat more parallel paths, a larger temperature difference

drives harder, and greater thickness spreads the same difference over a longer distance and so flattens the gradient. The conductivity  $k$  is the material's own

willingness to pass heat along, and it spans four orders of magnitude — copper near  $400 \text{ W/(m}\cdot\text{K)}$ , glass  $0.96$ , mineral wool  $0.04$ , still air  $0.026$ .

Run the numbers on a single window pane:  $1.5 \text{ m}^2$ ,  $3 \text{ mm}$  thick,  $20 \text{ K}$  from inside to outside.  $P = 0.96 \times 1.5 \times 20/0.003 = 9600 \text{ W}$ . Nine and a half kilowatts through one window. That is obviously wrong, and being clear about *why* it is wrong is the most useful thing this page can teach.

The equation is fine; the model is incomplete. Heat has to reach the glass from the room air and leave it into the outdoor air, and both of those handoffs are slow. The still-air film clinging to the inside surface has a thermal resistance of about  $0.12 \text{ m}^2\cdot\text{K/W}$  and the wind-scoured outside film about  $0.03$ , while the glass itself contributes only  $d/k = 0.003/0.96 = 0.0031$ . Resistances in series add, so the total is roughly  $0.155 \text{ m}^2\cdot\text{K/W}$ , giving  $U = 1/R = 6.5 \text{ W/(m}^2\cdot\text{K)}$  and a real heat flow of about  $195 \text{ W}$  — fiftyfold less than the bare slab calculation. The glass was never the bottleneck. This series-resistance picture is exactly the electrical analogy:  $\Delta T$  is voltage,  $P$  is current, and  $d/kA$  is resistance, which is why building science speaks in R-values ( $d/k$  per unit area) and U-values ( $1/R_{\text{total}}$ ) rather than in this equation directly.

**Worked example 6.3.** Given  $k = 0.96 \text{ W/(m}\cdot\text{K)}$ ,  $A = 2 \text{ m}^2$ ,  $\Delta T = 15 \text{ K}$ ,  $d = 4 \text{ mm}$ ; find  $P$  (heat flow rate).  $2 \text{ m}^2$  glass pane,  $4 \text{ mm}$ ,  $\Delta T = 15 \text{ K} \rightarrow 7200 \text{ W}$ .

## 6.4 Thermal Linear Expansion

(Physics 11-43)  $\Delta L = \alpha L_0 \Delta T$

Where,

$\Delta L$  = Change in length (m)

$\alpha$  = Linear expansion coefficient (1/K)

$L_0$  = Original length (m)

$\Delta T$  = Temperature change ( $^\circ\text{C}$ )

Nearly every solid grows when heated, and it grows by a fixed *fraction* of whatever length it already had. That is the whole content of  $\Delta L = \alpha L_0 \Delta T$ : the coefficient  $\alpha$  is the fractional growth per degree, so a long member moves more than a short one made of the same stuff. The reason solids

**So the standard error is treating one layer as the whole assembly.** Add layers by summing their resistances, never by averaging their conductivities, and never forget the two air films — in a well-insulated wall they are negligible, and in a window they dominate. It is also why a double-glazed unit works: the gain comes almost entirely from the trapped gas layer and the two extra surface films, not from doubling the glass.

Three further limits. This is a *steady-state* relation: it tells you the flow once the temperatures have settled and says nothing whatever about how long a wall takes to get there, which is a question of thermal mass and belongs to the diffusion equation. It is one-dimensional, which means it silently assumes heat goes straight through — a steel stud bridging an insulated cavity carries far more than its share of the area, and a thermal bridge can add a third to an assembly's real loss while the calculated R-value notices nothing. And  $\Delta T$  is a difference, identical in kelvin and Celsius, but imperial conductivities quoted in  $\text{BTU}\cdot\text{in}/(\text{hr}\cdot\text{ft}^2\cdot^\circ\text{F})$  are a different quantity with a different thickness convention buried in them, so convert deliberately rather than by feel.

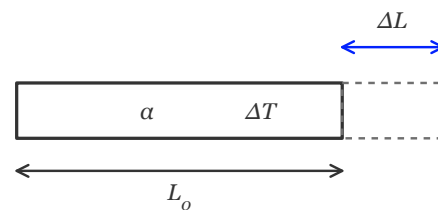


Figure 6.4: thermal linear expansion.

expand at all is a detail of the interatomic bond. The potential well an atom sits in is not symmetric — pushing two atoms together costs more energy than pulling them

apart by the same distance — so as the atoms vibrate harder, their *average* separation drifts outward. Expansion is that asymmetry, summed over every bond in the piece.

Take a 100 m copper riser in a hydronic building, filled at 20 °C and run at 82 °C. With  $\alpha = 16.5 \times 10^{-6}$  per K,  $\Delta L = 16.5 \times 10^{-6} \times 100 \times 62 = 0.102$  m. The pipe wants to be **102 mm longer** than it was when it was installed — a hand's width, in a shaft where the anchors are bolted to concrete. This is why risers get expansion loops, offsets, or bellows, and why the guides that keep the pipe pointing straight matter as much as the loops themselves.

The same coefficient handles areas and volumes to good approximation: an area grows at about  $2\alpha$  and a volume at about  $3\alpha$ , because the fractional growth applies in each dimension independently. Bond two metals with different  $\alpha$  back to back and the strip curves as it warms, which was the thermostat for most of the twentieth century. Go the other way and you get Invar, a nickel–iron alloy with  $\alpha$  near  $1.2 \times 10^{-6}$  — a tenth of steel's — for which Charles Édouard Guillaume took the 1920 Nobel Prize in Physics, an award for a material rather than a discovery, given because surveying and clockmaking needed lengths that did not care about the weather.

**Worked example 6.4.** Given  $\alpha = 12\text{e-}6$  1/K,  $L_0 = 10$  m,  $\Delta T = 50$  K; find  $\Delta L$  (change in length). *10 m steel beam ( $\alpha = 12\text{e-}6/\text{K}$ ) heated 50 K grows 6 mm.*

## Module 6 practice

**68.**  $Q = mc\Delta T$ , asked every way: A rooftop solar collector delivers 126 kJ of heat into a storage tank of water ( $c = 4200$  J/(kg·°C)), raising its temperature by 10 °C. *Determine the mass of water in the tank.*

**69.**  $Q = mc\Delta T$ , asked every way: A rooftop solar collector delivers 252 kJ of heat into a storage tank of water ( $c = 4200$  J/(kg·°C)), raising its temperature by 20 °C. *Determine the mass of water in the tank.*

**70.** Changing state: A block of ice sits in a warming tray at exactly 0 °C, and 1.5 kg of it melts to water — still at 0 °C. (L for melting ice: 334 kJ/kg.) *Calculate the heat the melting absorbed.*

**The largest mistake is applying this equation to a member that is not free to move.** A pipe anchored at both ends does not get longer; it develops stress instead. The stress is  $\sigma = E\alpha\Delta T$ , and for steel a 50 K rise gives  $210 \times 10^9 \times 12 \times 10^{-6} \times 50 = 126$  MPa — a serious fraction of the yield strength, and completely independent of length. That last point catches people: a short restrained member is under exactly the same stress as a long one, so shortening a run does not relieve anything. Continuous welded rail is laid under deliberate pre-tension for this reason, and a bridge without working expansion joints does not stretch, it buckles.

Two unit traps and one geometric one.  $\Delta T$  is a *difference*, so it carries the same number in kelvin and in Celsius — but not in Fahrenheit, where a 50 °F change is 27.8 K, and where a coefficient tabulated "per °F" is five-ninths of the per-kelvin value. Never substitute an absolute temperature for  $\Delta T$ . And  $\alpha$  itself is not constant over a wide range; handbook values are averages over a stated interval, and they drift at cryogenic and high temperatures. The geometric one: a hole in a heated plate gets *larger*, not smaller. Every dimension scales by the same fraction, including the empty ones, which is why warming a jar lid loosens it.

**71.** Changing state: A pot holds a rolling boil at a steady 100 °C while its element pushes 2260 kJ into the water. (L for vaporizing water: 2260 kJ/kg.) *Determine the mass of water boiled away.*

**72.** Warm it, then boil it: A camp kettle holds 1.5 kg of water at 30 °C. It is brought to a rolling boil, and then 0.75 kg of it boils away as steam. ( $c = 4200$  J/(kg·°C); L for vaporizing water: 2260 kJ/kg.) *Determine the heat for the climb to the boil, then for the whole job.*

**73.** Warm it, then boil it: An ice-maker takes in 2 kg of water at 15 °C, cools it to 0 °C, and freezes it solid. ( $c = 4200 \text{ J/(kg}\cdot\text{°C)}$ ;  $L$  for freezing water: 334 kJ/kg.) *Determine the heat removed in the cooling, then over the whole job.*

**74.** Heat on the move: A single-pane shop window of area 1 m<sup>2</sup> is 4 mm thick, and the glass ( $k = 0.96 \text{ W/(m}\cdot\text{°C)}$ ) holds 25 C° between its warm face and the street. *Calculate the rate at which heat leaks out through the pane.*

**75.** Heat on the move: A camping cooler's foam wall ( $k = 0.04 \text{ W/(m}\cdot\text{°C)}$ ) has a total area of 1 m<sup>2</sup> and is 20 mm thick. The summer air outside sits 25 C° above the ice inside. *Calculate the rate at which heat leaks in.*

**76.** The stretch of summer: The steel deck of a highway overpass runs 20 m at the winter design temperature. By the peak of a summer afternoon the deck is 50 C° warmer. ( $\alpha$  for steel:  $12 \times 10^{-6} / \text{°C}$ .) *Determine how much longer the deck becomes, in mm.*

**77.** The stretch of summer: A continuous steel rail runs 75 m between expansion gaps. From dawn to mid-afternoon the rail warms by 25 C°. ( $\alpha$  for steel:  $12 \times 10^{-6} / \text{°C}$ .) *Determine how much the rail grows, in mm.*

**78.** The Kettle: Last page of the paper. A kitchen kettle holds 2 kg of water at 40 °C, and its element is rated 1 kW. It is brought to a rolling boil, and then 0.6 kg boils away as steam. ( $c = 4200 \text{ J/(kg}\cdot\text{°C)}$ ;  $L$  for vaporizing water: 2260 kJ/kg.) Work each line — every answer feeds the next. *Determine the heat for the climb, the plateau, the whole job — and the minutes the element needs for all of it.*

**79.** The Kettle: Bonus mark, worked backwards. A 1 kW element brought 2 kg of water to the boil in exactly 7 minutes — and its owner wants to know how cold the tap ran that morning. ( $c = 4200 \text{ J/(kg}\cdot\text{°C)}$ .) *Determine the temperature the water started at.*

## Answer key

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|                         |                          |                 |
|-------------------------|--------------------------|-----------------|
| 1. 55 m                 | 28. 4 m/s                | 55. 60 s        |
| 2. 45 m                 | 29. 980 J                | 56. 125 V       |
| 3. 20 m/s               | 30. 50 kg                | 57. 6 A         |
| 4. 18 m/s               | 31. 1,960 J              | 58. 40 $\Omega$ |
| 5. 36 km/h              | 32. 7,840 J              | 59. 5 $\Omega$  |
| 6. 150 m                | 33. 200 W                | 60. 17.5 A      |
| 7. 20 m                 | 34. 5 m/s                | 61. 10 $\Omega$ |
| 8. 7 m/s                | 35. 80 %                 | 62. 3.75 kWh    |
| 9. 2 m/s <sup>2</sup>   | 36. 680 J                | 63. 720 W       |
| 10. -4 m/s <sup>2</sup> | 37. 90,000 J             | 64. 15 A        |
| 11. 12 m/s              | 38. 0.125 s              | 65. 8 A         |
| 12. 10 m/s              | 39. 5 Hz                 | 66. 30 $\Omega$ |
| 13. 2 m/s <sup>2</sup>  | 40. 3 m                  | 67. 2,400 W     |
| 14. 10 kg               | 41. 12 m/s               | 68. 3 kg        |
| 15. 98 N                | 42. 340.39 m/s           | 69. 3 kg        |
| 16. 147 N               | 43. 349.48 m/s           | 70. 501 kJ      |
| 17. 294 N               | 44. 0.45 m               | 71. 1 kg        |
| 18. 0.3 (no unit)       | 45. 0.7 m                | 72. 441 kJ      |
| 19. 450 N/m             | 46. 344 Hz               | 73. 126 kJ      |
| 20. 350 N/m             | 47. 0.5 m                | 74. 6,000 W     |
| 21. 40 N                | 48. 40 dB                | 75. 50 W        |
| 22. 98 N                | 49. 3.2 W/m <sup>2</sup> | 76. 12 mm       |
| 23. 200 N               | 50. 600 Hz               | 77. 22.5 mm     |
| 24. 0.3 (no unit)       | 51. 605 Hz               | 78. 504 kJ      |
| 25. 15 m                | 52. 343.42 m/s           | 79. 420 kJ      |
| 26. 50 N                | 53. 387 Hz               |                 |
| 27. 7 kg                | 54. 3 A                  |                 |